Zero-Totality in Action-Reaction Space: A Generalization of Newton's Third Law?

SABAH E. KARAM
Morgan State University, 1700 E. Cold Spring Ln.
Planning & Information Technology, Baltimore, MD, 21251
E-mail: Sabah.Karam@morgan.edu

In order to present the universe as a zero-totality the key concepts of nothingness and duality are required. Diaz and Rowlands introduce processes of conjugation, complexification, and dimensionalization using a universal alphabet and rewrite system to describe a physical universe composed of nilpotents. This paper will apply the concept of conjugation to the Newtonian duality action-reaction by introducing associated dual spaces called action space-reaction space. An attempt to generalize Newton's third law of motion, utilizing the concept of dual spaces, will follow in a manner suggestive of the zero-totality fermion-vacuum relationship.

Keywords: zero-totality, conjugation, duality, Newton's laws of motion, action-space, reaction-space.

1. Introduction

Diaz and Rowlands [1]-[2] have argued convincingly that the four fundamental parameters of physics are space, time, mass-energy, and charge. Superimposing charge on the other three parameters they identify a conserved and quantized Dirac state from which they derive the Dirac equation. The nilpotent version of the Dirac equation is derived from first principles and replaces the traditional matrix representation by an algebraic one. The Dirac algebra is applied to the baryon wavefunction, the strong interaction potential, and electroweak mixing allowing for a comprehensive treatment of the quantum field in terms of a quaternion-vector model. The 'wavefunctions' in the model are quaternion state vectors which are mathematical objects having a broader range of application than standard wavefunctions. The physical origin of quaternion wavefunctions lead to applications in particle physics not available in the conventional model [3].

The Dirac algebra incorporates three mathematical procedures called conjugation, complexification, and dimensionalization beginning in a zero-state and iterating to infinity. This paper will expand on conjugation, duality, and zero-state concepts used in the Dirac algebra in a non-standard attempt to generalize Newton's third law of motion. Conjugation and the zero-state depend on the context in which they are used. In an arithmetic setting numbers are conjugated by assigning alternating signs + and –, so that any number created is automatically paired with its opposite. Combining the two numbers under the operation of addition recovers the numeric 0. In a physical setting conjugation requires creation and annihilation operators. For any given elementary particle, say an electron, an antiparticle, the positron, is automatically created annihilating the created particle. Conjugation is then a process of bringing any element introduced into the system back to an original pre-defined zero-state of nothing. *Nothing* is not a scientific term even when referring to a vacuum, a region of space containing no matter, which still may contain gravitational fields. Zero is the starting point for a set of mathematical and physical rules used in constructing objects that preserve the original structures. Conjugation is a process that conserves a zero state. A negative charge, such as an electron, can only be part of the structure if a positive one is also part of the structure. A collision between the particle-antiparticle pair satisfies several conservation laws including the conservation law of electric charge in which the net charge before and after the collision is zero.

The concept of a zero-state appears in many areas of physics. It is a generally accepted theory in physics that the universe has zero (constant) total energy. The laws of thermodynamics (TD) can also be viewed as zero-state conditions ensuring that negative energy occurs in negative time and that any state satisfying TD laws are positive. Energy in isolated thermodynamic systems always remains constant. According to the 'big bang' and inflationary theory of the universe, matter and antimatter, as well as photons, are produced by the energy of the vacuum that was released following the phase transition. Matter consists of positive energy but is exactly balanced by gravitational energy thereby maintaining zero-energy of the universe.

The first claims concerning zero-total energy of an inflationary universe were made in the early 1960s. In the following years an idea that the universe may be a large-scale quantum-mechanical vacuum fluctuation emerged. It is theorized that positive mass-energy may be balanced by negative gravitational potential energy. By the turn of the twentieth century, researchers determined why the energy of a gravitational field is critical to the idea of a zero-energy universe. The gravitational property was used to show that the energy of a gravity field is truly negative. There are other cosmological arguments that matter-energy considered to be positive because it is increasing is exactly balanced by negative gravitational-energy resulting in a total-energy that is zero. An expanding universe extracts energy from the gravitational field, whereby more matter is created and hence more inertia [4]-[6]. Chemical reactions exhibit entangled zero-state properties as observed in redox reactions which involve a simultaneous transfer of electrons. Oxidation is a loss of electrons and reduction is a gain of electrons. Oxidation-reduction reactions are similar to acid-base reactions in which molecular, atomic, and ionic reactions are balanced with respect to electric charge. This also includes covalent bonding in which a stable balance of conjugate, attractive and repulsive, forces occurs.

A balancing act similar to that played out in gravity-matter interactions and redox reactions is evident in many conservation laws such as momentum, energy, mass, charge, and spin. Many mathematical equations describing physical events can be rewritten satisfying a zero-state condition that allows for new kinds of analysis and interpretations. Newton's laws of motion will be reexamined in light of Rowlands and Diaz's constraint that the physical universe "conserves" a Dirac state of space, time, mass and charge. Grouping the four parameters into a single mathematical expression creates a consistent and complete framework required needed to extend a physical reality from zero to infinity. In an attempt to generalize Newton's third law, "to every action there is an equal and opposite reaction," a key feature of the Dirac state will be employed, viz. the sum of all creation and annihilation operators maintains a zero-state. For fermionic fields this means that the positive frequency operators add a positive energy particle while the negative frequency operators annihilate a positive energy particle thereby maintaining a nothingness-state by simultaneously raising and lowering the energy. Inflationary universes and chemical balances conserve a zero state in a similar fashion.

2. Zero-Totality

Zero-totality conditions are generally satisfied in closed systems. Consider the conservation law of momentum where an interaction takes place between particles having masses m_1 and m_2 and traveling with velocities u_1 and u_2. If the final velocities are v_1 and v_2 then Newton's third law applies and we can write $m_1u_1 + m_2u_2 = m_1v_1 + m_2 v_2$. We rewrite this last equality in a form which underscores a zero state condition $m_1 (u_1 - v_1) + m_2 (u_2 - v_2) = 0$. If we let M represent the mass-vector (m_1, m_2) of the system, V the resulting velocity-vector $(u_1 - v_1, u_2 - v_2)$, and use dot product notation the last equation can now be written in the form $M \cdot V = 0$ allowing us to interpret momentum conservation in two ways. First, the mass-vector is orthogonal to the velocity vector and seond the zero-state is conserved. It appears that momentum plays the same role in classical mechanics that the *mean* plays in statistics and probability theory. In a statistical environment zero is conserved by the mean; the sum of the deviations of observations from the expectation value is 0. Squaring and summing deviations from the mean produces a positive value allowing us to create a new statistic called the standard deviation. The continuous Gaussian distribution can then be uniquely defined using two parameters: mean and standard deviation.

The probability density function of a quantum harmonic oscillator is an exact Gaussian distribution with second-degree terms. Squared terms enter into a description of wavefunctions and include an orthogonality condition. The nilpotent Dirac wavefunction incorporates several fundamental parameters including space, time, mass and charge, energy and momentum. Real and virtual components in the nilpotent state vectors also include parameters of mass and charge and *zitterbewegung* is understood to be a switching between charge and mass providing state vectors a supersymmetric status. The oscillation is interpreted as the quantum equivalent of real *action* followed by virtual *reaction*. A new kinematical model of the zitterbewegung phenomenon, describing the inertial property of varying electron mass as being a relation between momentum and velocity, suggests that mass is not only a "measure of self-interaction" but also a measure of the "coupling strength of the electron to its own field" [7]. Zitterbewegung may be caused by interference between positive and negative energy wave components. If this is the case, then it is a good example of action-reaction conserving the zero-state. We will return to this idea later in the paper. The motivation for developing a kinematical model is to explain the magnetic dipole field of the electron. The dipole field is viewed as an "*average*" over a zitterbewegung period and actual field is theorized to contain a high-frequency component oscillating with the zitterbewegung frequency and may be responsible for electron diffraction and Pauli exclusion [8].

The idea that zitterbewegung is a kind of averaging can be mathematically demonstrated using a Foldy-Wouthuysen transformation. Zitterbewegung terms vanish upon taking expectation values for wave-packets that are made up only of positive or exclusively negative energy waves. These observations highlight the importance of squared terms in physics but are not the main topic of this paper. Chapter 16 of *Zero to Infinity* (*ZtI*) [9] calls attention to how duality generated conjugations account for squared terms and why the factor 2 is found in so many, seemingly unrelated, physical phenomena. The techniques developed in *ZtI* provide mechanisms that permit the mathematical manipulation of zero and also eliminate the problems found in averages and orthogonality conditions. The hierarchy problem also vanishes. In fact, no infinities arise in the Dirac states as they do in the statistical calculations of quantum electrodynamics (QED) field theory, and renormalization is not required. The nilpotent vector representation of the Dirac equation is second quantized and self-dual by virtue of the operator and state vectors being identical (*ZtI*, p. 291).

There are other approaches to resolving complexity issues related to particle physics. The term dualism is used to characterize a general property of matter and has been used to explain one of the long-standing puzzles in the hierarchy of masses of the fundamental fermions [10]. The history of particle physics reveals that if a regular pattern is observed in the properties of matter, then it can be explained by invoking some underlying structures. An explanation for patterns of the fermion masses is based on a space-time dualism structure. Two reciprocal manifestations of space unified through an "inversion" region are used to address the complexity problem. The notion of dualism may be related to the ideas under discussion but will not be examined in this paper.

3. Zero-totalities are governed by dualities

Many have suggested that the "discovery of fundamental dualities is the result of all attempts at scientific investigation" (*ZtI*, p. 64). Maintaining a zero-totality universe relies on a duality concept that is dependent on a process called conjugation. The dependence that physics has on dualities has been observed by others and has been called thema-antithema. A historical study of theme-based dualities identified as *discreetness-continuum*, *projection-retrojection*, and *evolution-devolution* reveals that scientists typically have an allegiance to one of the themes (thema) while ignoring the conjugate theme (anti-thema) [11]. The original theme may be dominant but, failing to explain experimentally observed facts, requires an alternative theory called the antithema. The competition between "conjugate notions" of reality is not restricted to the field of physics but is found in many other disciplines as well. We will investigate disciplines with conjugated variables and duality models in the next section.

Dual-pairs, which negate each other, are required in creating something from nothing, the empty state, or the zero-state. The requirement is what is meant by conjugation. We will investigate this process in greater detail as we focus on the interdependence of duality and conjugation. In number theory the zero-state is a numeric 0. In set theory we start with the null-set as the empty state. We then create opposite pairs {+1, –1} and dual sets{S, ~S}. Under the operations of addition and union, respectively, the zero-state and empty state are conserved. Zero conservation can be also inferred from the law of conservation of momentum. Consider the physical system of an arrow in a stretched bow. The momentum of the system before releasing the arrow is zero. The momentum after releasing the arrow is zero. Both the arrow and bow acquire equal momentum but in opposite directions. The dual pair consists of equal momentum but in opposite directions, i.e. velocities of opposite values. Not only is Newton's third law of motion satisfied but zero is also conserved. The conjugation process in both the mathematical and physical examples is what is meant by duality. Duality preserves the total zero state, the 0 value, the null set, and the momentum. Starting with the idea of zero, or nothing, derived from dualities is a clue how to produce something from nothing. Physicists believe that the universe is composed entirely of fermion-antifermion interactions. The nilpotent Dirac equation mathematically describes the interaction and is itself based on a duality concept leading to an observation that "duality is not merely a 'component' of physics but an expression of the fundamental nature of physics itself" *(ZtI*, p. 443).

The Dirac equation is a relativistic generalization of the Schrödinger equation which can be written in a zero-state form satisfying conjugation and duality conditions. The novel mathematical formulation provides an insight into the nature of the Dirac equation and unifies a broad range of hypotheses about fermions and their interactions. Rewriting and factoring the relativistic equation for energy $E^2 = p^2 + m^2$ (by convention, c=1) a zero-totality condition

$$(\pm \boldsymbol{ik}E \pm \mathbf{ip} + 1\boldsymbol{j}m)\,(\pm \boldsymbol{ik}E \pm \mathbf{ip} + 1\boldsymbol{j}m) = E^2 - p^2 - m^2 = 0$$

emerges. In the context of Dirac's equation this is a nilpotent which squares to zero. The conjugate or dual state is the negative of the Dirac state $-(\pm \boldsymbol{ik}E \pm \mathbf{ip} + 1\boldsymbol{j}m)$ which is interpreted as being the rest of the universe or 'vacuum state' which is created simultaneously. Interactions between discrete charged objects occur because of the continuous vacuum state and agree with Mach's principle of inertial mass. It is extracted from a zero-totality superposition since it results in $(\pm \boldsymbol{ik}E \pm \mathbf{ip} + 1\boldsymbol{j}m) - (\pm \boldsymbol{ik}E \pm \mathbf{ip} + 1\boldsymbol{j}m) = 0$. The physical picture is a quantum mechanical universe created from nilpotent states within an infinite-dimensional Hilbert space. The 'universe' can be described as an entanglement of all possible nilpotent states and because it is zero-state balance between positive and negative charges it is electrically neutral. The Grassman algebra, used to represent the nilpotent states, is equivalent to the complex Hilbert space of modern quantum theory and represents a nonlocal superposition of fermionic (Dirac) states. The unique states also satisfy Pauli exclusion since fermions (electrons, neutrinos, quarks) with half-integer spin cannot occupy the same quantum state simultaneously. The universe is seen as being composed entirely of nilpotent fermionic or antifermionic wavefunctions. Boson wavefunctions are simply combinations of the two. Key insights into the physical world can be gained from purely mathematical models. The equations of Maxwell provided evidence that light is an electromagnetic phenomenon. The equation of Dirac implied the existence of a new form of matter called antimatter. Duality is central to a hypothesized inflationary universe in which energy is extracted from the gravitational field and produces matter.

A Dirac algebra is one in which the individual nilpotents $\Psi_n = (\pm \boldsymbol{ik}E_n \pm \mathbf{ip}_n + \boldsymbol{j}m_n)$ are wavefunctions satisfying the antisymmetric property $\Psi_n \wedge \Psi_m = -\Psi_m \wedge \Psi_n$, n≠m. We will rewrite this expression and reinterpret it the context of Newton's third law. We then use it in a heuristic argument ultimately connecting it to zero-totality. First let us rewrite the last property of nilpotents in the form $\Psi_n \wedge \Psi_m + \Psi_m \wedge \Psi_n = 0$ to call attention to zero conservation. The left side is understood as being a superposition of two fermion states having opposite spin and whose combined zitterbewegung can be viewed as a fermionic quantum dance or, euphemistically speaking, a virtual *jitterbug*. This description is consistent with zitterbewegung described as "*a new kind of bound state where the helical world line of one lepton is intertwined with the helical world line of another*" [7].

In the case n = m, we find a one-particle self-interaction in the Dirac state. The last equation can be rewritten in the form $2\ \Psi_n {}^\wedge \Psi_n = 0$. This equation can be interpreted as Pauli exclusion; each nilpotent wave function is unique, and the product of identical ones would zero the entire set of fermion states. An alternative way of looking at this last equation is to disregard the purely algebraic nature of the equation. We will replace the 0 on the right side of the equation with a Z in order to emphasize a physical zero-state condition. The new relationship is not an algebraic one, meaning that factor 2 does not cancel. The new formulation $2\ \Psi_n {}^\wedge \Psi_n = Z$ can be interpreted as electromagnetic self-interaction with the fermion bound to an oscillating electromagnetic field. This can be viewed as a de Broglie's pilot wave and is consistent with the observation that "*electron mass and spin can be identified with the energy and angular momentum of electromagnetic self-interaction*" [7]. A zero-totality condition is also satisfied.

We will now define action space as the space in which all discrete self-interacting fermions conserve the zero state, i.e., $2\ \Psi_1 {}^\wedge \Psi_1 + 2\ \Psi_2 {}^\wedge \Psi_2 + 2\ \Psi_3 {}^\wedge \Psi_3 + \ldots = Z$. The continuous vacuum is produced by the gravitational component and generates, for any given fermion, a state vector that is equivalent to $-1(\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ with negative energy. As a result any combination of a fermion and total vacuum produces a zero-state. The infinite superposition of all Dirac fermion states acts as the discrete Action Space for which a Reaction Space is generated, in this case the continuous gravitational vacuum. This transition from a discrete space to a continuous space, and vice versa, is the distinguishing feature of the Dirac algebra. The transition from discreteness to continuity is evident in many scientific theories and has been the source of seemingly contradictory notions of reality. Fermion states are incomplete without a vacuum and a supersymmetric partner. The discrete fermionic action space of kinetic energy requires a continuous reaction space vacuum of steady-state potential energy. The real fermion and the set of dual vacuum conjugate images combine to produce a conserved physical system. The system is conserved by a "simultaneously incorporating of both action space and reaction space of Newton's third law of motion" (*ZtI*, p. 149). We symbolically write Newton's third law in the form action + reaction = 0 inoorder to emphasize a zero-totality condition.

ZtI has presented novel mathematical techniques to analyze the Dirac equation having a coherent and integrated physical interpretation. The group symmetry for the Dirac algebra is almost identical to that for the Standard Model. Transitions in black body radiation from half to integral values are explained in terms of radiation reaction in which a transition is made between a discrete space and continuous space. The distinction between rest and relativistic masses becomes clear. Rest mass defines an isolated object having continuous kinetic energy. Relativistic mass, on the other hand, includes the effects of the environment. A photon, having no rest mass, acts as a classical potential energy term with discrete potential energy mc^2. Radiation pressure is produced by photon gas and the justification for action-reaction effects can be seen in the doubling of the value of the energy term that comes from the doubling of momentum when photons rebound from barriers, as in an absorption-emission process. The same thing happens in radiation reaction, a doubling of energy.

The mathematical constructs in *ZtI* treat discrete and continuous phenomenon simultaneously. In describing Dirac quantum states both continuous (fields, waves) and discrete (charges, particles) properties are incorporated. The results of Young's double-slit experiment are illustrative of the difficulties encountered in explaining quantum mechanics and originate in the discreteness-continuity dilemma. Reframing wave-particle duality in a nilpotent setting resolves the apparent dilemma.

4. Conjugation and duality concepts in other disciplines

Conjugation and dualities are indispensable in the Dirac algebra and can be found in almost every field of study including the natural sciences, philosophy, the humanities, social sciences, linguistics, and the medical sciences. Some paired terms are obviously dualistic, and identifying them is not difficult. These include *supply-demand*, *static-dynamic, thesis-antithesis, biotic-abiotic, thema-antithema*, *chaos-order*, and *intentional-unintentional.* There are dual pairs that, for the uninitiated, could never be recognized as properties that are in an oppositional relationship. The dual pairs *activation-inhibition, agency-structure, anabolic-catabolic, analog-digital, empirical-rational, glucagon-insulin, holistic-atomistic, ideal-real, immune-endocrine, inductive-deductive, integration-differentiation, anions-cations, osteoblast-osteoclast, nature-nurture, niche-biotope, oxidation-reduction, permittivity-permeability, reflective-reflexive, semantic-syntactic, systolic-diastolic* are part of this growing list.

It is a celebrated fact that duality is the central theme of quantum mechanics. Light exhibits both wave (field) and particle (matter) properties. The three hundred year old debate about the nature of light, dating back to the late 1600s when competing theories of light were proposed by Huygens and Newton, was finally resolved with the recognition that light has both wave and particle characteristics. Other disciplines have a similar history of evolving theories and witnessed the emergence of competing models [12]-[25]. The resulting standoff left no clear winner and there was final recognition that dual models were required to explain experimental results. A sampling of duality models from various disciplines sends a strong signal that duality and zero-totality may be as fundamental to other fields of study as they are to physics as the following examples will show.

A reflective-impulsive model has been introduced by social psychologists to explain cognition and behavior as a function of two interconnected mental faculties. Each faculty operates according to a different principle but interact in dualistic manner. The advantages of this dual-system model are "(a) their integrative power, (b) their foundation in well-established constructs of cognition and neuroscience; and (c) the ease with which they can explain the interplay of judgments and nonjudgmental processes" [26]. Dual cognitive science models identify paired voluntary-involuntary actions as conjugated variables.

The duality theory of production is a dual model that imposes a number of simplifying assumptions regarding economic production technologies, including dual concavity-convexity assumptions. Convexity is important in recovering technology information from economic models. Concave cost functions and convex profit functions are convex act as the conjugate functions in economic theories.

In optimization theory a dual problem is secondary problem with the property that its objective function is always a bound on the original, primal, mathematical problem. The same data are used in constructing the constraints in both the primary problem and the dual one. The coefficients in both problems are interpreted in a complementary manner, but the objective function reverses. If the objective function in the primary problem is to maximize a function, then the objective in the dual problem is to minimize the function. If the constraints in the primary model are constraints from above, then in the dual model, the constraints are constraints from below. Moving the conjugate constraints in the dual space corresponds to minimizing the slack between the original vector and the optimal vector.

An important property of set algebra, a principle of duality for sets, asserts that for any true statement about sets, the dual statement obtained by interchanging unions (⋃) and intersections (⋂) results in a true statement. A statement is said to be self-dual if it is equal to its own dual. DeMorgan's laws are transformation rules in which the Boolean operation of negation transforms the valid rule of conjunctions and disjunctions, into a complementary equally valid rule.

In electromagnetic theory and electrical engineering many properties are paired dualistically. These include *electric fields-magnetic fields, permittivity-permeability, piezoelectric-magnetostrictive, ferroelectric-ferromagnetic materials, electrets-permanent magnets, Faraday effect-Kerr effect, voltage-current, parallel-serial (circuits), impedance-admittance, reactance-susceptance, short circuit-open circuit, time domain-frequency domain,* and *conductance-resistance.*

The earliest use of a duality principle occurred in 1825 in projective geometry: Given any theorem in plane projective geometry, exchanging the terms "point" and "line" everywhere results in a new, equally valid theorem. Other examples include: duality in order theory, dual polyhedron and geometric duals.

In biology, dualism is the theory that blood cells have two origins, one from the lymphatic system and one from the bone marrow. The functions of chlorophyll and hemoglobin, inhaling and exhaling, muscle contraction and extension, anabolic and catabolic metabolism, osteoblasts and osteoclasts (bone creation-bone annihilation), lymphatic and myeloid elements are known to have opposite functions. The methods of category theory have been successfully applied to the investigation of what is referred to as the "basic dualism of biology" the phenotype and genotype of a given organism.

The wave-particle nature of light is a central concept of quantum mechanics. A less orthodox interpretation is the *duality condition* which is described by an inequality allowing wave and particle attributes to exist simultaneously. It is postulated that a stronger appearance of one of the attributes leads to a weaker appearance of the other meaning that the wave nature and particle nature can co-exist. A more recent experiment to be discussed in the last section of this paper demonstrates that wave and particle features can exist simultaneously.

In genetics, complementarity is the correspondence of DNA molecular components (nitrogenous bases) in the double helix, such that adenine in one strand is opposite to thymine in the other strand, and cytosine in one strand is opposite guanine in the other. This one-to-one relationship of the bases is called Chargaff's rule of base-pairing. Complementarity also means that one strand of nucleic acid (DNA or RNA) can pair with and serve as a template for its complementary strand.

In economics, two goods are considered to be complementary if the cross elasticity of demand is negative, as the price of one good increases the demand for the other decreases. Duality is the very foundation of a double entry book keeping system. Every transaction has a double (or dual) effect on the business as recorded in the accounts. For example, when an asset is bought another asset, cash (or bank), is simultaneously decreased or a liability such as creditors is simultaneously increased.

In neoclassical microeconomics duality refers to the existence, given appropriate regularity conditions, of indirect, dual, functions that embody the same essential information on preferences or technology as the more familiar direct, primal, functions such as production and utility functions. Dual functions contain information about both the optimal behavior and the structure of the underlying technology, or preferences, while the primal functions describe only the latter.

In molecular biology, code-duality refers to the fact that living systems always form a unity of two coded and interacting messages: the analog coded message of the organism itself and its re-description in the digital code of DNA. As analog codes, the organisms recognize and interact with each other in the ecological space, giving rise to a horizontal semiotic system (the ecological hierarchy), while as digital codes they (after eventual recombination through meiosis and fertilization in sexually reproducing species) are passively carried forward in time between generations. This, of course, is the process responsible for nature's vertical semiotic system, the genealogical hierarchy. Similar dichotomies are made in pathology, physiology, and in the diagnosis and treatment of medical problems. This philosophy centers on the ideas of the importance of maintaining the dynamic balance of these opposites for proper health. Investigations into the dynamic duality relationships between Symbiosis and Pathogenesis of microbial infections, which range along a continuum from conflict to cooperation, are considered a priority in the health community.

In cognitive psychology a duality arises naturally from the distinct roles played by a referent and a probe in comparative judgment. Dual-system models explain social cognition and behavior as a joint function of two interconnected mental faculties, each operating according to different principles. Dual terms such as: *reflective-impulsive, reflective-reflexive*, *propositional-associative, systematic-heuristic,* and *intentional-unintentional* are found primarily in social cognitive studies.

In learning theory, structure and function form a dualism. In the social sciences a dualism exists between human action and social structure. This issue has continued to divide sociologists. Symbolic interactionism stresses the active-creative components of human behavior and functionalism-structuralism the constraining nature of social influences on individual actions.

Opposing views of evolution-development (the evo-devo debate) exist over the roles and responsibilities assigned to such pairs as *structure-function, genes-environment, random-directed variations, innate-acquired characteristics, instructive-selective information, and self-organization-natural selection*. This debate is a result of self-referencing formulated in the ontic (internal-external) and epistemic (individual-local, population-global) considerations that are necessary to integrate developmental and evolutionary theories.

In the computational sciences, programming techniques are based on the concept of an "object" which is a data structure encapsulated with a set of routines that operate on the data. In object-oriented paradigms an object has dual characteristics: state and behavior. The following list depicts some of the dualistic categories used in software engineering: *digital-analog, synchronous-asynchronous, hierarchical-flat, static-dynamic, distributed-central processing, sequential-parallel, top down-bottom up, goal oriented-process oriented, explicit-implicit, active-passive,* and *predictable-statistical.*

The dualities between syntax and semantics and Fourier duality in algebraic theories are part of the same family of constructs. Abstract-concrete representations and theories are used to model the relationship between syntax and semantics.

Duality concepts can be appealed to when classifying academic disciplines. Some fields of study utilize variables that are continuous, while others require variables that are discrete. In the following list the first of each pair utilizes variables that take on continuous values or transformations and the second requires discrete ones: *calculus-statistics, analog computers-digital computers, ontology-epistemology, category theory-set theory, relativity theory-quantum mechanics, field theory-particle physics, and topology-geometry*.

Many engineering applications are characterized by combined discrete-continuous requirements and produce functions that are piecewise continuous. This is due to changes in modes of state variables or behavior. In chemical processes, for example, a discontinuity is associated with the beginning of a reaction. In materials engineering the phase change of elastic-plastic deformation represents a discrete change in the nature of the response of a mechanical system, even though the response is continuous.

5. The well-defined meaning of duality in *ZtI*

The Dirac nilpotent algebra is built upon clearly formulated duality concepts and emerges as a foundational approach to physics in a single structure. The mathematical structure created is a group of 64 conjugate variables composed of real-complex combinations of vectors and quaternions from which the nilpotent form of Dirac equation is derived. Conserved and non-conserved elements such as momentum-space are treated as conjugate pairs. The novel structure can be applied to classical, relativistic, and quantum systems. For example, the real-complex conjugate number systems allow transformations between the space-time of relativity theory and the electro-magnetic setting of Maxwell's equations. The equations of special relativity can be derived directly from the Dirac nilpotent. The two approaches to quantum mechanics are the discrete Heisenberg and the continuous Schrödinger. Each theory incorporates the missing feature in the other property when measurements in a real physical system are made. The wave-particle duality is a classic case of the continuous-discrete phenomenon. Making a distinction between the discreteness of matter and charge and the continuity of the vacuum allows for a reexamination of the entire domain of physical reality from a duality perspective. The Dirac algebra is one in which a conjugated nilpotent structure is preserved in a zero-totality dualistic environment. The classification of dualities in *ZtI* has two unique features not found in other disciplines. First, *ZtI* dualities can be uniquely categorized as being either discrete or continuous. Second, the variables relevant to each of the paired categories play a role in the environment of the other providing a mathematical and physical view from a dual perspective. The interaction (transformation) between a discrete space and a continuous space looks much like a harmonic oscillator in which the sum of the kinetic and potential energy is constant. The key feature of the discrete *action space*-continuous *reaction space* environment is to maintain zero-totality. The weak charge is a property unique to fermionic matter and the dipolar weak interaction yields a harmonic oscillator solution of the Dirac equation which is also typical of the components found in gaseous and condensed matter.

We note that the term *interaction* can be replaced with *transformation* without any loss of meaning. In mathematics the term 'transformation' means a rule or procedure describing a mapping (morphism) of one structure into another related structure. In set theory, mappings are functions; in algebra, they are linear transformations; in group theory, they are called are homomorphisms; and in topology, they are continuous functions. Transformations in this paper have been from an action space (e.g. Dirac fermion states) to a reaction space (e.g. vacuum state). The symmetry transformations found in the standard model of particle physics and central to the Dirac states in *ZtI* are: charge-symmetry (antiparticles replace particles), parity-symmetry (spatial variables are replaced by conjugates), and time-symmetry (direction of time is reversed).

The harmonic oscillator is a classic system such that when it is displaced from an equilibrium position experiences a restoring force. The solution of the Dirac equation depends on creation-annihilation operators for the production fermion-antifermion states which emerge from and disappear into the vacuum. The operators are essentially the same ones used in Quantum Field Theory. Zitterbewegung fluctuations in the vacuum are seen as the creation-annihilation of a weak dipolar fermion-antifermion pair via a harmonic oscillator creation-annihilation mechanism. The same fluctuations are responsible for the Casimir or Van der Waals force. The action of the weak dipole moment is seen as the cause of the creation of fermion-antifermion combinations from the vacuum and also as the cause of mutual annihilation. *ZtI* emphasizes that physical variables and phenomena are at their roots conjugated and dualistic and discrete-continuous properties are superimposed in a Dirac nilpotent algebra. The first term in each pair of the following examples from *ZtI* is associated with entities having a discrete nature, the second continuous: *particles-waves, Heisenberg-Schrödinger equations, quantum mechanics-wave mechanics, QED theory-SED theory, inertial mass-gravitational mass, matter-gravitational field, discrete matter-filled vacuum, potential energy-kinetic energy, charged particles-mass-energy, space dimension-time dimension, Leibnizian differentials-Newton fluxions, countable numbers-uncountable numbers,* and *fermion state-vacuum state.* Mathematically formulated nilpotent states in *ZtI* accomplish a seemingly impossible task of simultaneously treating discrete and continuous variables. This novel mathematical treatment allows for a reinterpretation of physical reality from a dualistic perspective. In the new framework gravity is a considered to be a continuous source which is responsible for the linkage of infinitely many discrete Dirac nilpotent states of which the universe is composed.

Inertia can be understood in this new framework as an interaction between matter in discrete space and the gravitational vacuum in a continuous space. Continuity implies *instantaneous interaction* whereas discreteness requires a *quantum interface*. This can consider a new approach to uniting general relativity (GR) and quantum mechanics (QM). The key difference between older quantum gravity (QG) theories is that in *ZtI* quantum-gravity is replaced with quantum-inertia. Gravitational mass and inertial mass are equivalent but the former, being continuous, and the latter, being discrete, means that it is inertia rather than gravity that should be quantized. Treating gravity as a continuous instantaneous force eliminates the need for a quantized attractive force rather than a fictitious repulsive force.

The dichotomization of variables, equations, states, physical properties, and mathematical operations offers a new approach to overcoming dilemmas of measurement and observation and has implications beyond the realm of physics. The implications for the foundations of physics are profound when we consider that a region of space, whether continuous or discrete, is the domain of interaction of material objects as well as fields. Newton's laws are concerned with objects and forces and the rules governing their interaction. In the context of the fermion space-vacuum space interaction Newton's third law of motion is one that connects discrete and continuous forces. Newton's first law of motion, on the other hand, applies to bodies with zero velocity and bodies moving with constant non-zero velocity. The first law can also be stated more generally in terms of force. If the net force, i.e., the sum of all vector forces, is zero then the velocity of the object is constant. The concept of zero conservation (of forces) is relevant here. The symbol *Z* was introduced in Section 3 to symbolize a numeric zero. We will now expand the meaning of zero-totality, represented by Z, to signify not only an algebraically manipulated numerical value, but also an object at rest, an object moving with constant velocity, an object experiencing zero total net force, an equilibrium point, a stability condition, an adiabatic condition, a steady state condition, a symbiosis, a balance, a symmetry condition, homeostasis, etc.

Appealing to the definition given to Z we will generalize Newton's first law of motion and call it the **Law of Zero-Totality**. This law applies to any field of study built on a duality model and conjugated variables. We now state a generalization of Newton's third law of motion as **Action + Reaction = Z.** The physical environment in which action and reaction occurs will be called Action Space-Reaction Space. The action space-reaction spaces in *ZtI*, for example, are the Dirac states of discrete fermions and the continuous vacuum. The predominant function of the fermion action space- vacuum reaction space is to accommodate the harmonic oscillation solutions of the Dirac equation and their zitterbewegung fluctuations.

6. Action Space-Reaction Space

The notion of *space* is not new in physics. Utilizing Fourier transforms quantum mechanics can be formulated using momentum space wavefunctions or particle space, sometimes called real space, wavefunctions. In this section we will construct phenomenological spaces in which to view Newton's third law. Newtonian methods have been used effectively for over two hundred years in generating formulas needed for space travel and a rationale for wearing seat belts. We know that the laws of conservation of energy, momentum, and angular momentum have more general validity than Newton's laws. The laws apply to both light and matter as well as to classical and quantum physics. Until recently [27] it has been impossible to demonstrate in a single experiment both a continuous and discrete property of light. That is why the double-slit experiment is such a paradox. Properties of light appear to be a particle in one experimental setting and a wave in another. *ZtI* resolves many paradoxes by focusing on the continuity-discreteness duality.

> Lorentz invariance arises solely from the mathematical description of the parameters in terms of real or imaginary numbers but the result of this forced union is that, *for the purpose of measurement*, either a quantity which is continuous must become discrete, or a quantity which is discrete must become continuous. The first is the particle option (time and mass become discrete), while the second leaves us with waves (space and charge become continuous) (*ZtI*, p. 223).

Many paradoxes materialize when making a transition from continuous states to discrete states or vice versa. Newtonian fluxions are the mathematical precursor of the differential calculus. Newton was concerned with velocities and acceleration and he defined velocity as "*the ultimate ratio of evanescent quantities*." These evanescent quantities appear in the definition of velocity as an instantaneous rate of change resulting in a ratio that appears as the undefined ratio 0/0 originating in the limit $y'(x_0) = \lim \Delta y/\Delta x$. Newton's definition of velocity is a mathematical procedure linking discreteness and continuity. The definition is valid for well-defined and differentiable functions. A continuous limiting process over real numbers, resulting in a discrete number, resolves many of Zeno's paradoxes but is nevertheless puzzling and contrary to common sense.

The methods of the differential calculus were introduced by Newton to describe unseen earthly and astronomical forces, falling objects and orbiting planets. They appeared at the time when two divergent views about the nature of light were being debated. The interplay between continuous and discrete objects and forces amplified the paradoxes. *ZtI* has revealed the true nature of the dilemma and has been the motivation for defining action spaces-reaction spaces where interactions between the continuous and discrete can be studied. The key paradigms in the theory of light, special and general relativity, and quantum mechanics may be nothing more than an issue related to continuous-discrete interactions.

> Continuous vacuum energy, such as we would expect from a 'filled' vacuum, is what we mean by nonlocality. It is the continuing connectedness, through the vacuum, of apparently discrete fermionic states, and it is required to maintain Pauli exclusion. Rest mass is a localization and therefore discretization of the continuous total vacuum energy (*ZtI*, p. 510).

The interactions typical of vacuum-fermion are the primordial spaces in which Newton's third law functions. The continuous space-discrete space duality is the most logical environment for Newton's third law of motion to operate. The links between the continuous and discrete physical domains, between the changing and the fixed, between the real and imaginary, and between the orderable and nonorderable can now be described. The continuous-discrete duality is apparent in the distinction between potential and kinetic energies, which is also a distinction between conserved-nonconserved quantities, or fixed-changing conditions. The duality may also be expressed in terms of the distinction made between space like theories of Heisenberg approach and the time like theories of Schrödinger. In the quantum mechanics-stochastic electrodynamics duality, we see not only a discrete-continuous dichotomy but also a real-imaginary one.

We can represent symbolically the action space-reaction space zero-totality conditions of many of the examples discussed above using a new symbol (+) to represent the interaction between continuous spaces and discrete spaces.

potential energy (+) kinetic energy = Z,
inertia (+) gravity = Z,
relativistic push (+) newtonian pull = Z, and
fermions (+) vacuum = Z.

The interaction between fermion space and vacuum space means that the discrete fermionic self-interaction (action space) makes a field contribution force to the continuous vacuum (reaction space). The weak van der Walls force causing the fermion-vacuum interaction and the oscillation between the two spaces, in which zero-totality is maintained, is the generalized setting of Newton's third law. Aggregation of matter, referred to as *complexity* in *ZtI*, originates in the harmonic oscillation of interacting fermion-vacuum spaces. This zitterbwegung activity produces subatomic, atomic, chemical, molecular, organic, non-organic, planetary, galactic, and ultimately cosmological structures and is responsible for complexity (aggregation).

The quantum delayed-choice experiment referred to earlier in which 'strong nonlocal correlations were observed" showed that the photon behaved simultaneously as a wave and a particle [27]. If conservation laws are also satisfied then we can unambiguously write

particle (+) wave = Z.

There have been other attempts at refuting Bohr's principle of complementarity that both wave and particle behaviors can be exhibited in the same experiment. A double-slit experiment initially performed at Harvard University in 2001 claimed that the Englert–Greenberger duality relation was violated but that linear momentum was conserved [28].

7. Conclusion

Competing wave-particle theories about the dual nature of light took centuries to resolve with an ultimate realization that both theories are correct. The theories describe dualistic or, as some say, complementary features of the same phenomenon. No foundational principles, however, have ever been proposed that address the resulting paradoxes of the dual theories. That is why the relativity theory-quantum mechanics debate is now reaching its one-hundredth year. The roots of the debate go back to the 16th century but under a new name. There have been attempts, by string theorists and others, to unify GR-QM into a theory of everything. M-theory and quantum gravity, however, are nothing more than mathematical appendages on the original theories and they introduce unyielding singularities and infinities. *ZtI* has not only uncovered the roots of the debate but has provided the mathematical machinery for accommodating conflicting methodologies and interpretations. We have a description of elementary particles that is consistent with the Dirac equation, special relativity, and quantum mechanics. This is precisely the kind of foundational approach needed to understand and resolve issues presented in the original accounts. A full accounting of relativity in the context of quantum mechanics can be found in *ZtI*.

Galilean inertia was codified in Newton's first law of motion. Both laws find their ultimate expression in Einstein's special theory of relativity. Newton defined inertia to mean "the innate force possessed by an object which resists changes in motion." Inert objects are resistant to change especially if they are inanimate. Laws that apply to orbiting planets are not very easily transported into the life sciences. We may be able to apply, however, the generalized form of Newton's first and third laws to other fields in which duality models exist without compromising the theoretical framework in which living matter and forces operate. The following relationships are constructed from the examples in Section 4. They suggest a way of viewing dual-model theories in other disciplines in a generalized Newtonian framework.

As a first example we look at the reflexive-compulsive model in social psychology which can be written in the form

Voluntary (+) Involuntary action = Z, or
Reflective (+) Reflexive thought = Z

The symbol (+) represents the interaction between the conjugate dual modes of thought. The vacillating (oscillatory) nature of reflective-reflexive thinking is well known. According to the dual system model the reflective system depends on symbolic thought and is the basis of conscious experiences. Conscious experiences can only be communicated in a discrete symbol dependent manner whereas reflexive thought is a continuous process.

The Sym-Pat focal point duality which models the dynamic continuum from cooperation to conflict can also be reformulated in the formalism of zero-totality as

Symbiosis (+) Pathogenesis = Z.

For the conjugate variables in evolutionary theory we may write the relationship that describes the interaction between genes and the environment

Nature (+) Nurture = Z.

In molecular biology there is a fundamental distinction made the inheritance of traits and evolution. Biologists are interested in knowing to what extent an organism's phenotype affect its genotype. The duality models that have been proposed appear to satisfy a zero-totality condition

Genotype (+) Phenotype = Z.

In an examination of the epistemological and logical terminology problem found in language [29], educational philosophers introduced a new term to describe concepts related to dualisms and dualities. The new term, *distinctions*, implies a dualistic interaction, and there are over one hundred pairs revealed in the treatise. It is argued that self-action, interaction, and transactions are responsible for the loss of precision and efficiency in the use of words. Conjugate terms are introduced having precise meanings to remedy the terminological crises prevalent in language today.

Rowlands and co-collaborators (Diaz, Cullerne, Koberlein, Marcer, Mitchell, Schempp, Hill) have made a strong case for the foundational role of dualities in physics. Other theories incorporating a *duality principle* may find it appropriate to apply the generalizations of Newton's laws presented in this paper. Biological systems create order and destroy order by extracting, storing, and transmitting information in a continuous fashion. There are two fundamental, yet opposing, forces that compete with each other in all bio-systems. They are comparable to the creation and annihilation operators of fermion states. One force leads to uniformity, over ecological time and the other leads to diversity over evolutionary time. Chapter 19 of *ZtI*, co-authored with biologist Vanessa Hill, demonstrates that the geometric and algebraic structures required in particle physics also apply equally to replicating biological systems. Translation, transcription, replication, the genetic code and the grouping of amino acids are known to be vitalized by the same fundamental processes that drive the elementary particles. DNA, RNA, and the genetic code efficiently process information and can be represented by mathematical structures, both algebraic and geometric. The surprising development is that these same mathematical nilpotent structures are used to describe nature at atomic and sub-atomic scales!

There is a plethora of seemingly divergent *principles of duality* in the literature. Complementarity, if no longer true about the wave-particle phenomenon, is thriving in the academic world where disciplines co-exist and produce terminological tsunamis after each earth-shaking discovery. The author, in the spirit of the times, has contributed a new term to the wave of terminological confusion by coining a new term called the *Dualogy Principle*. The new principle attempts to accommodate theories in which duality models have clearly defined conjugated variables and/or transformations [30]. Restating the principle in the language of the generalized forms of Newton's first and third laws we write

> Dualogy Principle: In order to describe a physical or a non-physical event co-dependent dual conjugated variables defining the state and/or transformations that define the behavior of the phenomenon are required. The state variables satisfy the Law of Zero-Totality and the transformations satisfy Action + Reaction = Z.

It should be noted that Descartes dualism and Bohr complimentarity do not satisfy the Dualogy Principle; in each the conjugated states are irreconcilable or independent.

The original statement of the dualogy principle had relevance in a recent study on the living cell. Experimental results have led to a proposal by Sungchul Ji that the Gibbs free energy levels of enzymes in living cells are quantized with discrete units of information and energy [31]. Conversion (transformation) of information to energy is possible. Maxwell's demon, a being that violates the second law of thermodynamics, is replaced by a being that "receives energy and dissipates heat into his environment while performing his molecular work" then Maxwell's angel, can carry out the assigned work without violating the second law. The dual responsibility, *taking in energy-dissipating heat* simultaneously, is one performed by molecular machines. The two fundamental entities (conjugate variables) which operate in all molecular machines have been identified as information and energy. A dual model of the living cell is presented along with dual theories called *the conformon theory of molecular machines* and the *cell language theory of biopolymer interactions*. Conformons store mechanical energy in the form of Gibbs free energy and genetic information and are responsible for goal-directed molecular actions driving the cells. The new theory about the molecular properties of cells treats the interaction between energy and information and may be amenable to treatment within a zero-totality framework. The fields of thermodynamics and information theory appear to contradict the theory of evolution. If biologists look at the contradictory notions of order and disorder as a dualistic interaction of action-reaction spaces then it may be reasonable to consider that their interaction also satisfies zero-totality and can written in the form

> Entropy (+) Evolution = Z.

The action space in which evolution occurs has been identified as space-time [32]. It is reasonable to call the reaction space of entropy temperature-space.

The primary purpose of this paper is to derive a generalized form of Newton's third law of motion from the concept of zero-totality. It quickly became clear that such a generalization only made sense if Newton's first law was viewed not as a law of motion or inertia but rather as a law regarding forces. Many of the arguments that have been presented in this paper have admittedly been heuristic, but the profound nature of the foundational issues raised and answered in *ZtI* justifies this heuristic approach. An attempt has been made to apply generalized Newtonian laws to fields of study outside of physics. Duality models in other disciplines have paradoxical characteristics reminiscent of the wave-particle description of light. In *ZtI* the three mechanisms of conjugation, complexification, and dimensionalization are used to describe fundamental mathematical and physical processes in general and to derive the Dirac equation in particular. The processes of conjugation, complexification, and dimensionalization are all considered to be nothing more than "alternative forms to duality" (*ZtI*, p. 13). The nilpotent form of the Dirac equation can be represented in a compact 5-vector form with mass as the fifth dimension on equal mathematical footing with the four dimensions of spacetime. An extra fifth dimension also appears in a variant of the Kaluza-Klein model unifying gravity and electromagnetism called the induced matter theory. The form includes all of the conjugated variables found in classical and quantum physics. State information about one of the conjugate pairs can be exchanged for state information about the other. The nilpotent equation permits statements about conservation in action space to be transformed into equivalent statements about nonconservation in reaction space, and vice versa. The Dirac nilpotent is self-dual and being the square root of zero produces two solutions simultaneously. Boson states are generated from fermions and, in general, the nilpotent algebra produces a doubling effect and the reversal of properties. Discreteness is produced from continuity; a conserved state is created from a nonconserved state; and, in general, a conjugated state is paired with a nonconjugted one. *ZtI* provides a unique foundational approach to physics providing a rigorous mathematical framework which can handle incompatible concepts like discreteness and continuity simultaneously. The "*mathematics of duality*" maps onto the mathematics of reality and is a serious contender for a theory of everything. The second main purpose of this paper is to argue that zero-totality and duality apply to many other academic disciplines and have relevance far beyond the scope of particle physics itself.

REFERENCES

1. P. Rowlands, "The fundamental parameters of physics," *Speculat. Sci Tech*., 6, 69-80 (1983).
2. B. Diaz & P. Rowlands, "A universal alphabet and rewrite system," arXiv:cs/0209026, 23 September 2002.
3. P. Rowlands, "The nilpotent Dirac equation and its applications in particle physics," arXiv:quant-ph/0301071. 15 Jan 2003
4. R.P. Feynman, **Lectures on Gravitation** (Addison-Wesley, Reading, 1962-3).
5. A.H. Guth, **The Inflationary Universe: The Quest for a New Theory of Cosmic Origins**, (Persus, Cambridge, 1998).
6. S. Hawking, **The Universe in a Nutshell**, (Bantam, New York, 2001).

7. D. Hestenes, "Quantum Mechanics from Self-Interaction," *Found. Physics*, 15 (1), 63-87 (1985).
8. D. Hestenes, "Zitterbewegung Modeling," *Found. Physics*, 15 (1), 63-87 (1993).
9. P. Rowlands, **Zero to Infinity: The Foundations of Physics**, (World Scientific, Singapore, 2007).
10. V. Yershov, "Dualism of spacetime as the origin of the fermion mass hierarchy," *Apeiron*, 12 (1), 1-25 (2005).
11. G. Holton, **Thematic Origins of Scientific Thought: Kepler to Einstein**, (Cambridge: Harvard Univ. Press, 1988).
12. E. Andrade, "The Interrelations between Genotype / Phenotype / Environment: A Semiotic Contribution to the Evo-Devo Debate," (2002). http://www.library.utoronto.ca /see/SEED/Vol5-2/Andrade.pdf
13. S. Birkett & P.H. Roe, "The mathematical foundations of bond graphs. II: Duality," *J. Franklin Inst.* 326 (5), 691-708 (1989).
14. S. Birkett & P.H. Roe, "The mathematical foundations of bond graphs. III: Matroid theory," *J. Franklin Inst*. 327 (1) 87-108 (1990).
15. T. Cheon & t. Shigehara, "Fermion-Boson Duality of One-Dimensional Quantum Particles with Generalized Contact Interactions," *Phys. Rev. Lett.* 82, 2536-2539 (1999).
16. C.A. Drossos, "Structures, Points, and Levels of Reality." G. Sica (Ed.). **Essays on the Foundations of Mathematics and Logic**. (Monza, Italy: Polimetrica International Scientific, 2005), p. 83-114.
17. U. Eco, **Semiotics and the Philosophy of Language**, (Indiana University Press. Bloomington, 1984).
18. B. Englert, "Visibility and Which-Way Information: An Inequality,"*Phys. Rev. Lett*. 77 (11), 2154-2157, (1996).
19. J. Macnamara & G.E. Reyes, **The Logical Foundation of Cognition** (London: Oxford Univ. Press. 1994)
20. E.M. Melhado, **Jacob Berzelius, The Emergence of his Chemical System**. Madison; The University of Wisconsin Press, 1984).
21. K.R. Popper, **Studies in the Philosophy of Biology**, (University of California Press, Berkeley, 1974).
22. R. Rosen, **Life Itself**, (Columbia University Press, New York, 1991).
23. R. Rosen, "The representation of biological systems from the standpoint of the theory of categories," *Bull. Mathematical Biophysics*, 20, 317-341 (1958).
24. A. Sfard, "On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin," *Educational Studies in Mathematics*, 22 (1), 1–36 (1991).
25. S.-H. Huang, W. Zhou, & A. Jong, "Focal Point Theory Models for Dissecting Dynamic Duality Problems of Microbial Infections," *Journal of Biomedical Biotechnology*. doi: 10.1155/2008/856314. (2008).
26. R. Deutsch, "Duality models in social psychology: From dual processes to interacting systems," *Psychological Inquiry*, 17 (3), 166-172 (2006).
27. A. Peruzzo, P. Shadbolt, N. Brunner, S. Popescu, & J.L. O'Brien, A quantum-delayed choice experiment. Science, 338 (6107), 634-637 (2012).
28. S.Afshar, "Violation of the principle of complementarity and its implications," arXiv:quant-ph/0301071. 7 Jan 2007
29. J. Dewey & A. F. Bentley, ***Knowing and the Known***, (Boston: Beacon Press, 1949).
30. S.E. Karam, "Dualogy: A category-theoretic approach to duality," Second International Conf. on Complementarity, Duality, and Global Optimization in Sci. and Eng., (2007). http://dualityscience.com/yahoo_site_admin/assets/docs/cdgo2007_PDF.21362207.pdf
31. S. Ji, **Molecular Theory of the Living Cell: Concepts, Molecular Mechanisms, and Biomedical Applications**, (New York: Springer, 2012), p. 389.
32. M. W. Ho, "Towards a theory of the organism." *Integrative Physiological Behavioral Science*, 32, 343-363 (1997).