Dualogy: A Category-Theoretic approach to Duality

Sabah E. Karam
Sabah@dualityScience.com

Presented May 2007 at the Second International Conference on Complementarity, Duality, and Global Optimization in Science and Engineering
http://www.ise.ufl.edu/cao/CDGO2007/

Abstract: Duality is ubiquitous. Almost every discipline in the natural and social sciences has incorporated either a principle of duality, dual variables, dual transformations, or other dualistic constructs into the formulations of their theoretical or empirical models. Category theory, on the other hand, is a purely mathematically based theory that was developed to investigate abstract algebraic and geometric structures and especially the mappings between them. Applications of category theory to the mathematical sciences, the computational sciences, quantum mechanics, and biology have revealed structure preserving processes and other natural transformations. In this paper conjugate variables, and their corresponding transformations, will be of primary importance in the analysis of duality-influenced systems. This study will lead to a category-based model of duality called Dualogy. Hopefully, it will inspire others to apply categorification techniques in the classification and investigation of dualistic-type structures.

Keywords: duality theory, optimization, category theory, conjugate, categorification.

1 INTRODUCTION

Duality arises in linear and nonlinear optimization techniques in a wide variety of applications. Current flows and voltage differences are the primal and dual variables that arise when optimization and equilibrium models are used to analyze electrical networks. In economic markets the primal variables are production and consumption levels and the dual variables are prices of goods and services. In structural design methods tensions on the beams and modal displacements are the respective primal and dual variables.

Notions of duality arise in the natural and social sciences, in engineering and especially in the humanities. There is a proliferation of terms that connote a property of dualness. Terms such as *bifurcation, bisimulation, bivalence, complementary*, *conjugate*, *dialectic*, *diametric*, *dichotomy*, *dipole*, *dual*, *dualism*, *duality*, *equivalence, invariance, inverse*, *opposite*, *parity*, *polarity*, and *symmetry* all include in their definition the idea of an interaction or partnership between two distinctly identifiable entities having both properties (state variables) and behavior (functionality). Mathematical models can be classified in set theory by the set of variable states determined by the parameters in the model and by the sets of previous and future states of the variables. Category theory, on the other hand, focuses on the structural and functional relationships of the mappings in the mathematical models.

Duality is sometimes defined as the quality or character of an event, an action, a state of being, a thought, or any other real or abstract entity that is characterized by two mutually exclusive but jointly conjugated parts. It is also defined as a correspondence between apparently different theories that lead to the same physical results. The observer/observed, position/momentum, and time/energy variables in quantum mechanics are said to be conjugate with respect to each other such that a perturbation of one of the variables results in the conservation of the other variable. Conjugate variables are said to be dual to each other since their roles can be interchanged by a change of a frame-of-reference resulting in another notion of duality called symmetry. In duality the concept of 'sameness' is inherent and cannot be avoided. In set theory sameness means equality but when we discuss transformations and mappings an idea of equivalence enters the picture.

Applying category-theoretic constructs to disciplines other than mathematics is nothing new. It has been successfully applied to Quantum Field Theory [4], Molecular Biology [36], and Computer Science [5]. The term categorification, introduced in 1994, refers to the process of finding category-theoretic analogs of ideas phrased in the language of set theory. The analogy between set theory and category theory has the following equivalence: elements ↔ objects, equations involving elements ↔ object isomorphisms involving objects, sets ↔ categories, functions ↔ functors, equations involving functions ↔ natural isomorphisms involving functors. An isomorphism is simply a one-to-one mapping between two sets that preserves the relationship of elements under corresponding operations on each sets. A more complete description of categories will be given later. Details about categorifiction can be found in [3]. Categorification refines the concept of 'sameness' inherent in duality and makes a sharp distinction between isomorphism and equality. In this paper we will be utilizing category theory to analyze duality concepts across several disciplines. This exercise will result in a cross-categorification effort that will be called Dualogy.

2 DUALITY ACROSS THE DISCIPLINES

In the following paragraphs a survey of some major fields of study that incorporate a dual concept in the axioms and theories will be listed along with a short description of their respective duality related principles and/or terminology.

In **optimization theory,** a dual problem is another problem with the property that its objective function is always a bound on the original mathematical problem, called the primal. The same data is used in constructing the constraints in both the primary problem and the dual one. The coefficients in both problems are interpreted in a complementary manner, but the objective function reverses. If the objective function in the primary problem is to maximize a function, then the objective in the dual problem is to minimize the function. If the constraints in the primary model are constraints from above, then in the dual model, the constraints are constraints from below. Moving the constraints in the dual space corresponds to reducing the slack between the candidate vector and the optimal vector.

Optimization methods include several specialized types of duals having specific constraints. They are: *Dorn's dual, Fenchel's Conjugate dual, Generalized penalty-function/surrogate, Geometric dual, Inference dual*, *Lagrangian Dual,* and *Linear Programming (LP) Dual* (the cornerstone of duality). In canonical form: Primal: Min { cx: x >= 0, Ax >= b }. Dual: Max { yb: y >= 0, yA <= c }, *Self Dual* (when a dual is equivalent to its primal), *Semi-infinite dual, Superadditive dual, Surrogate dual, Symmetric dual,* and *Wolfe's dual.*

An important property of **set algebra**, called the principle of duality for sets, asserts that for any true statement about sets, the dual statement obtained by interchanging unions and intersections, interchanging U and Ø, and reversing inclusions also results in a true statement. A statement is said to be self-dual if it is equal to its own dual.

In **electronics**, the word dual refers to two devices or two circuits having mathematical descriptions that are identical except that voltages in one formula correspond to currents in the other formula. For example:

- a capacitor is the dual of an inductor;
- resistance is the dual of conductance;
- two resistance-devices in series are dual to two conductance-devices in parallel;
- electrostatic motors are dual to magnetic motors; and
- Kirchoff’s current law and voltage law are dual.

In **electromagnetic theory** and **electrical engineering** properties are paired in dualistic relationships, they include the following among them:

- electric fields and magnetic fields
- permittivity - permeability
- piezoelectricity - magnetostriction
- ferroelectric - ferromagnetic materials
- electrets - permanent magnets
- Faraday effect is dual of the Kerr effect.
- voltage - current
- parallel - serial (circuits)
- impedance - admittance
- reactance - susceptance
- short circuit - open circuit
- time domain – frequency domain

The differential equations developed for DC circuit analysis can be applied to AC circuit analysis by replacing the real numbers with complex numbers.

In **mathematics**, duality has a variety of interrelated meanings. Dualities translate mathematical structures in one theory into another mathematical structure in another theory in a one-to-one fashion.

- The earliest use of a duality principle occurs in 1825 in projective geometry: Given any theorem in plane projective geometry, exchanging the terms "point" and "line" everywhere results in a new, equally valid theorem. Other examples include: De Morgan dual in logic, duality in order theory, dual polyhedron, and geometric dual.

- In category theory, the objects of one theory are translated into objects of another theory and the morphisms (structure-preserving function) between objects in the first theory are translated into morphisms in the second theory, but with direction reversed. Using a duality of this type, every statement in the first theory can be translated into a *dual* statement in the second theory, but the direction of the arrows is reversed. Examples of this type include: dual spaces in linear algebra, Pontryagin duality, which relate abelian groups to other abelian groups, Tannaka-Krein duality, and the Stone duality, which relate Boolean algebras to topological spaces.

- Dual graphs allow us to turn coloring maps into coloring the vertices. Given a planar graph G, a dual graph of G has graph vertices each of which correspond to a face of G and each of whose faces correspond to a graph vertex of G.

In **biology,** dualism is the theory that blood cells have two origins, from the lymphatic system and from the bone marrow. The functions of chlorophyll and hemoglobin, inhaling and exhaling, muscle contraction and extension, anabolic and catabolic metabolism, osteoblasts and osteoclasts, lymphatic and myeloid elements are known to have opposite functions. Category theory has been used to represent neural networks and has been utilized in the investigation of what is referred to as the “basic dualism of biology” the phenotype and genotype of a given organism [1].

In **physics,** experiments have been conducted demonstrating that light and matter exhibit properties of both waves and particles. It is a central concept of quantum mechanics. The debate over the nature of light and matter dates back to the 1600's when competing theories of light were proposed by Huygens and Newton. A less orthodox interpretation is the *duality condition*, described by an inequality [16] which allows wave and particle attributes to co-exist, but postulates that a stronger manifestation of the particle nature leads to a weaker manifestation of the wave nature and vice versa.

The term duality also refers two different types of physical systems represented by equations that are identical. Comparing a mechanical harmonic oscillator with an RLC circuit reveals that the differential equations describing their behavior are the same. An RLC circuit is an electrical circuit composed of a resistor (R), an inductor (L), and a capacitor (C). The equation for the charge Q in the circuit is

$$LQ'' + RQ' + Q/C = 0$$

The equation for the velocity of a mass on a spring, with damping, looks very much like the RLC circuit.

$$mx'' + bx' + kx = 0$$

The correspondence between the dual variables are:

- The position, x, corresponds to charge Q.
- The spring constant, k, corresponds to the electrical elastance, 1/C, the reciprocal of capacitance.
- The damping factor, b, corresponds to electrical resistance, R.
- The mass, m, corresponds to the inductance, L.

In **genetics,** complementarity is the correspondence of DNA molecular components (nitrogenous bases) in the double helix such that adenine in one strand is opposite to thymine in the other strand and cytosine in one strand is opposite guanine in the other. This one-to-one relationship of the bases is called Chargaff's rule of base-pairing. Complementarity also means that one strand of nucleic acid (DNA or RNA) can pair with and serve as a template for its complementary strand. Complementary strands are related by the rules: A pairs with T or U and C pairs with G.

In **economics,** complementarity is a concept similar to that of externality. Two goods are complements if their cross elasticity of demand is negative. That is, as the price of one good increases, the demand to the other good decreases. Duality is the very foundation of double entry book keeping system and it follows from the fact that every transaction has a double (or dual) effect on the position of a business as recorded in the accounts. For example, when an asset is bought, another asset cash (or bank) is simultaneously decreased. A liability such as credit is simultaneously increased.

In neoclassical **microeconomics** duality refers to the existence, given appropriate regularity conditions, of indirect, dual, functions that embody the same essential information on preferences or technology as the more familiar direct, primal, functions such as production and utility functions. Dual functions contain information about both the optimal behavior and the structure of the underlying technology, or preferences, the primal functions describe only the latter.

In **philosophy,** dualism is a theory stating that mental phenomena are not reducible to physical phenomena. Both are wholly distinct making up two unique realms of being. Dualism is a philosophical theory of mind associated with the philosopher Descartes according to which human beings are constituted by two distinct metaphysical substances or realms: Thought and Extension. Dualism sometimes takes the form of *cosmic dualism*. Examples include Platonic philosophical dualism, which divided the world into the realm of ordinary human sense-experience, and the world of intellectual thought. There is also anthropological dualism, which sees human experience as divided between the flesh and the spirit.

In ancient Chinese **philosophy** the yin/yang concept is used to account for changes in the universe in a comprehensive way. According to the theory, all phenomena consist of two opposing aspects along with their implicit conflict and interdependence. In traditional Chinese **medicine,** the imbalance of yin and yang lead to disease and medical problems. In anatomy the human body is divided into the internal region (yin) and external region (yang). Similar dichotomies are made in pathology, physiology, and in the diagnosis and treatment of medical problems. The philosophy centers on the ideas of the importance of maintaining the dynamic balance of these opposites for proper health, the evolution of events as a process, and acceptance of the inevitability of change.

In **molecular biology,** code-duality refers to the fact that living systems always form a unity of two coded and interacting messages, the analog coded message of the organism itself and its re-description in the digital code of DNA. As analog codes the organisms recognize and interact with each other in the ecological space, giving rise to a horizontal semiotic system (the ecological hierarchy), while as digital codes they (after eventual recombination through meiosis and fertilization in sexually reproducing species) are passively carried forward in time between generations. This of course is the process responsible for nature's vertical semiotic system, the genealogical hierarchy.

In a book on the philosophy of **language,** a linguist describes duality as a structure from which language derives its meaning. Life and death, mind and matter (dualism of Descartes), space and time, subsistent forms and spacio-temporal objects, subjective ideas and objective reality, good and evil, yin and yang [14]. Examples of severa other pairs are included.

In the **humanities** John Milton is referred to as the “Poet of Duality" [36].

In **psychology** a duality naturally arises from the distinct roles played by a referent and a probe in comparative judgment. Current ongoing investigations are attempting to explain how reference duality and referential duality are dualistic themselves.

The **general theory of relativity**, published by Einstein in 1916, is a geometrical theory that postulates that the presence of mass and energy causes space-time to be curved and that this curvature affects the path of free particles and even deflects the path of light. In classical mechanics gravity was regarded as a force. In General Relativity, however, gravity is not considered to be a force but is, rather, a consequence of the curvature of space-time.

Modern physicists have reduced forces to four basic ones: the gravitational force, the electromagnetic force, the strong nuclear force, which binds the protons together in the nucleus of the atom, and the weak nuclear force, which causes spontaneous radioactive decay in atoms. In 1979 the theories of electromagnetic and weak interactions were combined into the electroweak theory. Attempts to unify all known forces began in earnest with Maxwell’s successful 19th century unification of electricity with magnetism. Einstein’s equivalence and relativity theorems extended Maxwell's unification attempts. The effort to unify all known forces into a comprehensive theory, is called the Grand Unification Theory (GUT). While some success has been made in unifying the gluon force, between quarks, with the electroweak force problems arise when the force of gravity is included. In order to accommodate gravitational forces a new theory, called string theory, was developed in 1996. It is a theory of elementary particles that incorporates relativity and quantum mechanics. The particles are viewed not as points but as extended objects called **strings**.

String theory is the only known approach to resolving the GUT problem that is at the core of modern physics: the incompatibility of quantum mechanics and gravity. Moreover, the most important physical principles, gauge theory and general relativity, are predicted by string theory. It is also a realization of an old dream that physics, at the fundamental level, should be determined by mathematical principles alone with no arbitrary dimensionless parameters. Duality is a cornerstone of the current understanding of string theory. Duality allows different descriptions of the same theory, some very simple and classical and others highly complicated and quantum mechanical. Dualities have taught us that superstring theories are in fact the weakly coupled descriptions of a single theory in various regimes. While duality as a phenomenon is not specific to string theory; in string theory duality is pervasive. The various dualities discovered in string theory have led to numerous advances in quantum field theory (the exact solutions of some supersymmetric gauge theories), mathematics (Calabi-Yau mirror symmetry), and quantum gravity (the counting of black hole entropy). The essential meaning of duality, for string theorists, is hence a correspondence between apparently different theories that lead to the same physical results.

In **quantum field theory,** the fermion-boson duality [9] describes a phenomenon of two systems, with distinct point interactions related by coupling inversion, that share an identical spectrum with symmetric and antisymmetric states interchanged [17]. Maxwell demonstrated the equivalence of electricity and magnetism. Einstein demonstrated the equivalence of matter and energy in the famous formula $E = mc^2$ and spent most of his life trying to find, what is commonly called today, a Theory of Everything (TOE). Einstein believed that the final task of physics was to unify general relativity (gravitational space-time) and electromagnetism. Current mainstream thinking requires that a TOE unify the four fundamental forces of nature (gravity, the strong nuclear force, the weak nuclear force, and the electromagnetic force) and also explain the spectrum of elementary particles. There has been progress toward a TOE in unifying electromagnetism and the weak nuclear force in an electroweak unified field theory and in unifying all of the forces except for gravity that in the present theory of general relativity is not a force. The missing piece in a theory of everything involves combining quantum mechanics and general relativity into a theory called quantum gravity.

One of the most surprising results in **quantum mechanics** is the entanglement of two or more distance particles. Even though there are still questions regarding the fundamental issues of quantum theory, quantum entanglement has started to play an important role in engineering applications. Quantum information processing, quantum metrology, quantum imaging and quantum lithography have utilized entangled (paired) photons, also called biphotons or two-photon states, in both fundamental and applied research. Quantum teleportation and communication of biphotons does not violate the limit established by speed of light transmission of electromagnetic waves. Communication between the paired particles is instantaneous!

There have been recent arguments for the case of duality by particle physicists . Dualism is known to be a general property of matter. It is used to explain one of the long-standing puzzles in the hierarchy of masses of the fundamental fermions [42]. The history of **particle physics** shows that if a regular pattern is observed in the properties of matter then it could be explained by invoking some underlying structures. An explanation for the pattern of the fermion masses is based on the conjecture of *space-time dualism* in the form of two reciprocal manifestations of space unified through an “inversion” region. Physicists are considering a dualistic view of space-time to explain seemingly contradictory interpretations of quantum effects. Space-time dualism is also a basis for studying complexity and hierarchical systems in organisms.

Duality is also viewed as **a bridge between physics and philosophy**. According to one physicist there are five distinct facets of duality in physics, viz. reciprocity, parallelism, alternative formulation, synthesis, and measurement incompatibility. A table highlighting the dual partners, with several examples for each category, can be found in the refernce [31].

In **learning theory** [37] the concepts mathematical structure and mathematical functions form a dualism. In the **social sciences** [11] a dualism exists between human action and social structure. The issue has continued to divide sociologists. Symbolic interactionism stresses the active, creative components of human behavior and functionalism, structuralism, and Marxism emphasize the constraining nature of social influences on individual actions. Attempts to overcome duality for a grand unified theory, acceptance of complementarity, or accepting contradictory theories are the three options.

In **theology,** the Koranic verse 51:49 states that “all things are created in pairs.” In the Islamic world the concept of duality (zawgyne is used colloquilly to refer to the pair husband/wife) is an ontological axiom of worldly existence and applies to the present life as well as to the afterlife. The Koranic principle of duality states explicitly that complementary or dual characteristics exist *a priori.* In the Islamic world duality is observed in all of creation, physical and metaphysical, animal and vegetable.

Opposite views of **evolution/development** (the evo/devo debate) exist over the roles and responsibilities assigned to such pairs as structure/function, genes/environment, random/directed variations, innate/acquired characteristics, instructive/selective information, and self-organization/natural selection. This debate results from the self-reference present in the ontic (internal/external) and epistemic (individual-local/population-global) considerations that are necessary to integrate developmental and evolutionary theories.

In the **computational sciences** programming techniques are based on the concept of an “object” which is a data structure encapsulated with a set of routines that operate on the data. In object-oriented (OO) paradigms an object has a dual characteristic: state and behavior. The following table depicts some of the dualistic categories used in the software engineering dichotomies.

- digital - analog
- synchronous - asynchronous
- hierarchical - flat
- distributed - central-processing
- sequential - parallel
- static - dynamic
- top down – bottom up
- goal oriented – process oriented
- explicit - implicit
- goal oriented – process oriented
- active - passive
- predictable - statistical

The duality between **syntax and semantics** and Fourier duality for algebraic theories are part of the same family of constructs. Considering abstract and concrete representations, we can regard theories in their relationship to models what syntax is to semantics.

This is a less than exhaustive list of duality principles across the disciplines. We will revisit many of the duality relationships described above from the perspective of category theory and commutative diagrams.

3 CONJUGATE VARIABLES AND DUALITY

Closely related to duality concepts is the idea of conjugacy relations. Conjugate means means joined together especially as a pair. Newton's third law of motion, for every action there is an equal and opposite reaction, identifies the action/reaction pair. In mechanics the sum of a conjugate pair, potential and kinetic energy, is constant. In thermodynamics the internal energy of a system is expressed in terms of pairs of **conjugate variables** such as pressure/volume, temperature/entropy, stress/strain, and chemical potential/particle number pair. Noether's theorem, discovered in 1915, holds for all physical laws based upon the action principle, which essentially allows the reformulation of the differential equations of motion into equivalent integral equations. The evolution of the physical system from one state to another is realized by requiring that the action be minimized, the most important consideration in optimization methods. Noether's theorem is a relationship between pairs of conjugate variables; if the action is invariant under a shift in one of the two physical variables, then the equations of motion resulting from holding that action stationary conserves the value of the other of the pair of variables.

Conjugate pairs have played a crucial role in quantum mechanics. The Heisenberg uncertainty principle is a mathematical property of a pair of canonical conjugate quantities. The principle mathematically limits the accuracy with which it is possible to measure such pairs. Applying the uncertainty principle to the position/momentum of any object, it implies that if we continue increasing the precision with which position is measured then the measurement of the momentum will be less and less precise, and vice versa.

The most common observables, canonical conjugate quantities, satisfying the uncertainty principle are:

- position, x, and momentum, p, satisfying $\Delta \mathbf{x} \; \Delta \mathbf{p} \geq h/4\pi$, h = Planck's constant
- time coordinate, t, and energy, E, satisfying $\Delta \mathbf{t} \; \Delta \mathbf{E} \geq h/4\pi$
- angular position and angular momentum,
- two orthogonal components of the total angular momentum operator of an object,
- the cosmological constant, Λ, and space-time volume.

The phrase, conjugate variables, can also be applied to the primal/dual pair in optimization analysis. Maximizing the primal problem results in the minimization of the dual problem.

4 CONJUGATE TRANSFORMATIONS AND DUALITY

A fundamental conclusion of the Heisenberg Uncertainty Principle is that no physical phenomenon can be described as a classic point particle or as a wave but, rather, a wave/particle duality best describes the phenomenon. The transformation of the particle into a wave and vice versa is what gives rise to the uncertainty principle. The transformations that are characterized by a change from one type of behavior to another will henceforth be called a conjugate transformation, in direct analogy and correspondence to conjugate variables. The wave/particle duality can therefore be characterized, not only, by a pair of conjugate variables, but also, by a pair of conjugate transformations.

The term 'transformation' in general means a rule describing the conversion of one syntactic structure into another related syntactic structure. In geometry, a transformation is any of a variety of different operations, such as rotations, reflections and translations. In genetics a transformation is the process by which the genetic structure of a cell is changed when foreign DNA is transferred into its cells.

All known forces have associated particles (photon, W-particle, gluon, graviton) and display behavior similar to that observed in electromagnetic phenomena, viz. transformations from the particle to the wave state, and vice versa. The famous equation of special relativity $E = mc^2$ establishes the transformational relationship between energy and mass. The conjugate variables, energy and mass, have corresponding conjugate transformations matter to energy.

These are analogous to the conjugate variables, position and momentum, and conjugate transformation, particle to wave.

5 CATEGORY THEORY

Category theory focuses on morphisms (transformations) that transmit structure. Category theory is a general mathematical theory of structures and systems of structures. It allows mathematicians to see how structures of different kinds are related to one another. The theory is philosophically relevant. It is considered by many as being an alternative to set theory as a foundation for mathematics and can be thought of as constituting a theory of concepts. As set theory was being formulated at the end of the nineteenth century paradoxes revealed that mathematics had a disturbingly shaky foundation. With the aim of placing set theory, in particular, and mathematics generally on a firmer logical pedestal, Hilbert and others looked to Euclidean geometry for a model. Until that time the objects of mathematical attention had been quite specific: real numbers, complex numbers, curves, and surfaces. Something more general was sought this time. Hilbert commented, "If among my points I consider some systems of things (e.g., the system of love, law, chimney sweeps ...) and then accept only my complete axioms as the relationships between these things, my theorems (e.g., the Pythagorean) are valid for these things also." It is interesting to note that Hilbert's insight suggested a category theoretic perspective more than 40 years before its creation. In other words, the nature of the objects being investigated is irrelevant. Truth was banished and replaced by provability. The new axiomatic spirit was to consider *structures*, viz. arbitrary sets equipped with operations that obeyed certain rules.

Category theory was introduced by Einlenberg and Mac Lane [15] in 1945 in connection with algebraic topology, a branch of mathematics in which topological spaces are studied using techniques of abstract algebra. Initially, the notions were applied in topology, especially algebraic topology, as part of the transition from homology (an intuitive and geometric concept) to homology theory, an axiomatic approach. In the paper, an axiomatic formalization of the relation between structures and the processes preserving them is given.

The study of duality in convex and non-convex systems is extensive. Optimization techniques, game theory, economic science, theoretical physics, chemistry, mathematical programming, variational analysis, nonconvex-nonsmooth analysis, and critical point theory are just a few of the areas of application [9]. Duality underlies many aspects of *extremum principles* in natural systems including geodesics, minimal surfaces, Karush-Kuhn-Tucker (KKT) optimality conditions, harmonic and subharmonic maps. Equilibrium states of many field equations are all critical points of functionals defined on some appropriate constraint set or manifold. Category theory has also been used as a theoretical tool in the study of optimization techniques [18].

The classic example of a category is *Set*, the category with sets as objects and functions as morphisms (which can be viewed as transformations in the general sense), and the usual composition as composition. Well-known examples of categories include:

- *Diff* - smooth manifolds are objects, smooth maps are morphisms
- *Group* - groups are objects, homomorphisms are morphisms
- *Ring* - rings are objects, ring homomorphisms are morphisms

- *Top* - topological spaces are objects, continuous functions are morphisms
- *Vect* - vector spaces are the objects and linear maps are the morphisms

FORMAL DEFINITION OF CATEGORY
A category, ***C***, is a class of objects, **Ob *C***, together with a class of morphisms, **Mor *C***, between objects. The morphisms satisfy the following properties:

1) For every two objects X, Y in **Ob *C***, there is a subset, Hom(X, Y), of **Mor *C***. These subsets are all pairwise disjoint. When first referring to an element, f, of Hom(X, Y), we will usually adopt the usual function notation: f: $X \longrightarrow Y$

2) For every X, Y, Z in **Ob *C***, and morphisms f: $X \longrightarrow Y$ and g: $Y \longrightarrow Z$, there is a morphism h: $X \longrightarrow Z$ called the composition of g with f. Again, we will adopt the usual functional notation: h = g ○ f.

3) Composition is associative, that is, for f: $W \longrightarrow X$, g: $X \longrightarrow Y$, and h: $Y \longrightarrow Z$, h ○ (g ○ f) = (h ○ g) ○ f.

4) For every object, Y, there is a morphism **1** in Hom(Y, Y) such that for all f: $X \longrightarrow Y$ and g: $Y \longrightarrow Z$, **1**○f = f and g○**1** = g. This morphism is called the identity.
Now suppose we have morphisms f: $X \longrightarrow Y$, f ': $X \longrightarrow W$, g: $W \longrightarrow Z$, and g': $W \longrightarrow Z$ such that g ○ f ' = g' ○ f. That is, g (f (X)) = g' (f ' (X)) = Z. We summarize this by saying that the following diagram commutes.

Commutative Diagram

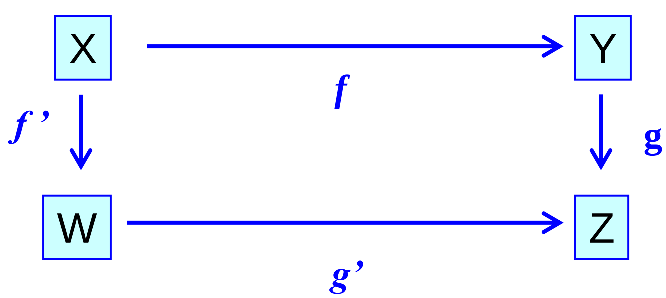


Figure 1 – A diagram is called commutative if any two chains of arrows have the same start and end categories

6 COMMUTATIVE DIAGRAMS ACROSS THE DISCIPLINES
The previous sections demonstrate that duality constructs exist in many disciplines but with rather disparate meanings. We will now examine dualistic concepts, in relation to the conjugate variables and conjugate transformations (morphisms), across several disciplines using category theory diagrams. The construction of these commutative diagrams will be the heuristic leading to propositions of Dualogy.

Our first example is from optimization theory. The primal-dual problems can be viewed as objects spaces in category theory. Let X be the set on which the primal problem is defined and let $\leq$ be a partial order relation. We now define the dual problem with a new partial order relation $\leq_{new.}$ The definition the dual problem is

$$x \leq_{new} y \text{ if and only if } y \leq x.$$

Linear Programming

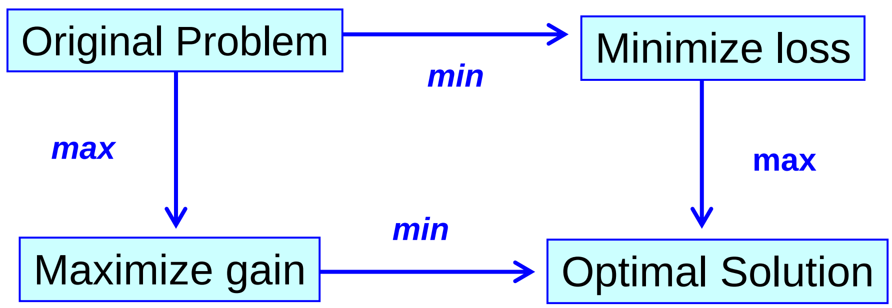


Figure 2 – Minimizing a max is equivalent to maximizing a min

World Religions

The commutativity of spiritual and material values

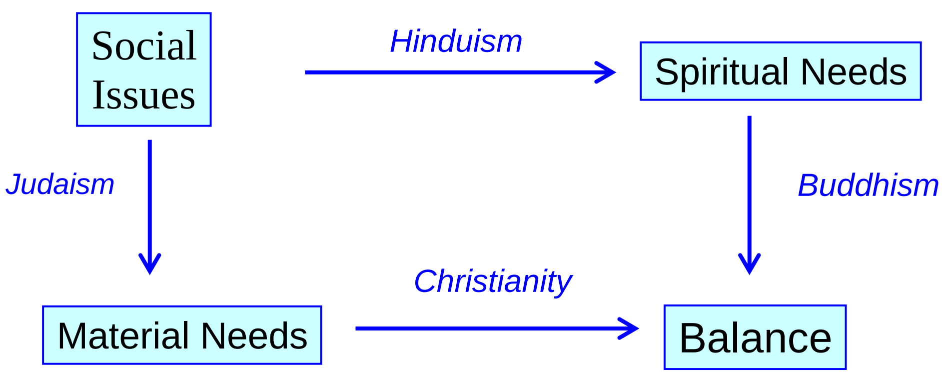


Figure 3 – Hinduism is to Buddhism as Judaism is to Christianity

Heisenberg Uncertainty Principle

Wave-particle nature of photon in double-slit experiment

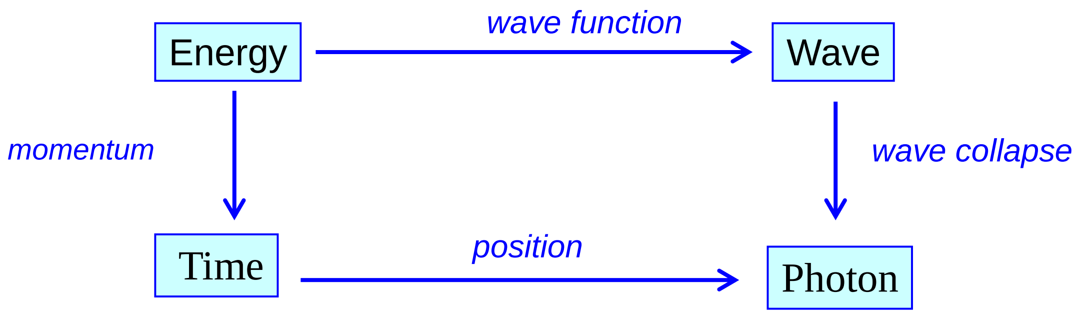


Figure 4 – Particle is to wave as position is to momentum

Formal Languages

Object language vs. metalanguage

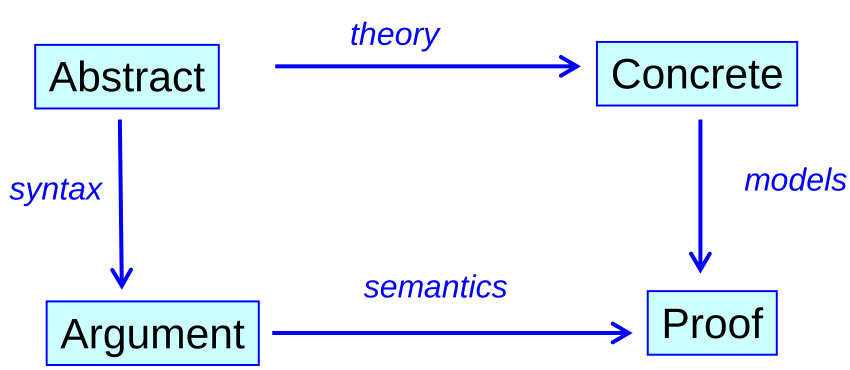


Figure 5 – Syntax is to semantics as theory is to models

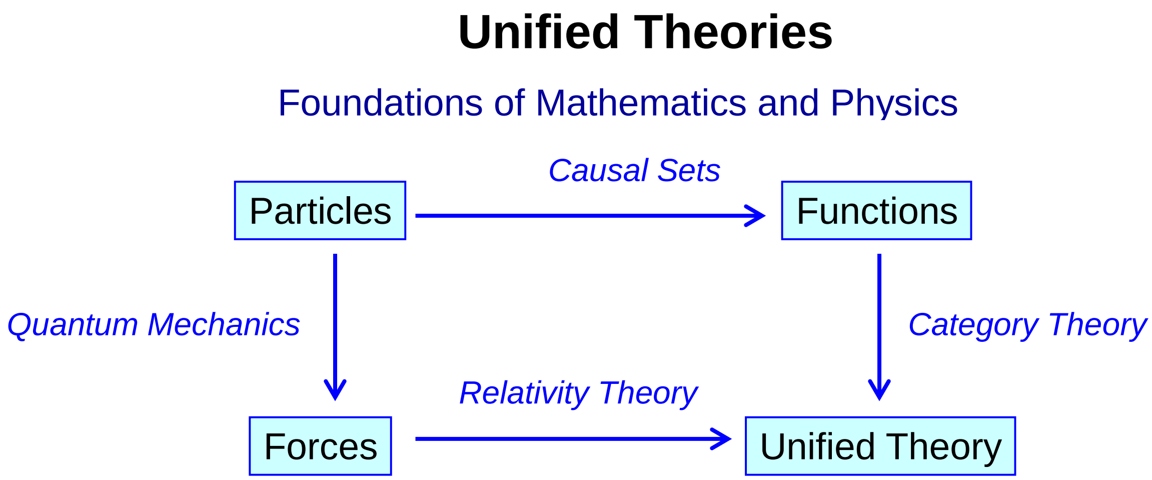


Figure 6 – Set theory is to Category Theory as Relativity Theory is to Quantum Mechanics

Category theory originated in the study of abstract mathematical systems but has now yielded profound insights in computer science, music [27], quantum mechanics, biology and psychology. Applications in the last two fields are in the early stages of development. In psychology the Macnamara's project is the categorization of the relationship between perception and cognition by means of functors, between *the category of gestalts* (the domain of perception) and *the category of kinds* (the domain of cognition) leading to the observation that "*category theory is to psychology as calculus is to physics"* [28]. A sheaf theoretic and categorical formulation of consciousness with applications to neural networks, knowledge, and cognition has also been developed [23].

CIRCULAR DIAGRAMS AND SECOND-ORDER CYBERNETICS

Cybernetics is an interdisciplinary study connecting such diverse fields as neuroscience, network theory, logic modeling, and control systems. It is a study of processes that feed back into themselves. Circularity conditions have led to deep conceptual problems in science. Russell's paradox is the most famous of the logical or set-theoretical paradoxes. But circularity, if modeled correctly, can help us to understand fundamental phenomena, such as self-organization, goal-directedness, identity, and life. Circular processes are observed in complex, networked systems such as biological organisms, ecologies, economic, and other social structures. Second-order cybernetics emphasizes autonomy, self-organization, cognition, and the role of the observer in modeling a system [20].

Cybernetics has a dual focus. The first is the way in which a 'system' is described as a discrete set of components whose functional interrelationships define the system as a whole. The other focus of concern is processes such as dynamics, history, and evolution. A system was best explained in terms of how the effects of its actions, outputs, circled back, as inputs, to influence, not only, the system's state, but also, its subsequent actions. It should be noted that category theory can be, but presently has not been, applied to the functionality aspects of cybernetics. In this context bond graphs, used for modeling and simulating the dynamic behavior of physical systems, should also be mentioned. Bond graphs are labeled di-graphs whose edges are called bonds. The edges represent the bilateral signal flow of the power-conjugate variables effort and flow [7]. Physical systems are commonly constrained to the behaviors that satisfy the basic principles of physics (conservation of energy, continuity of power, and positive entropy) and domains that are each characterized by a conserved quantities (called conjugate variables in this paper). Bond graphs, along with their reversible transformations, and cybernetics, along with feedback controls, are useful tools in depicting self-referential systems and commutative diagrams.

Circularity occurs in common place nature-driven events such as the four seasons and the hydrologic cycle, but also in axiomatic systems such as set theory, relativity theory, and quantum mechanics. Circularity is important in analyzing dualities and is fundamental in the conversion of commutative diagrams to circular diagrams that can be modeled in second-order cybernetics. A formal definiton of circularity will now be given that mimics the definition of commutative diagram.

Definion of Circularity: Four morphisms, f,g,h, and k, satisfy the circularity condition if

$$\mathbf{k} \circ \mathbf{h} \circ \mathbf{g} \circ \mathbf{f} = \mathbf{1}\text{, where } \mathbf{1} \text{ is the identity morphism.}$$

$$f\colon X \longrightarrow Y,\quad g\colon Y \longrightarrow Z,\quad h\colon Z \longrightarrow W \quad\text{and}\quad k\colon W \longrightarrow X$$

If f, g, h, k are defined by the mappings then we can draw the circular diagram:

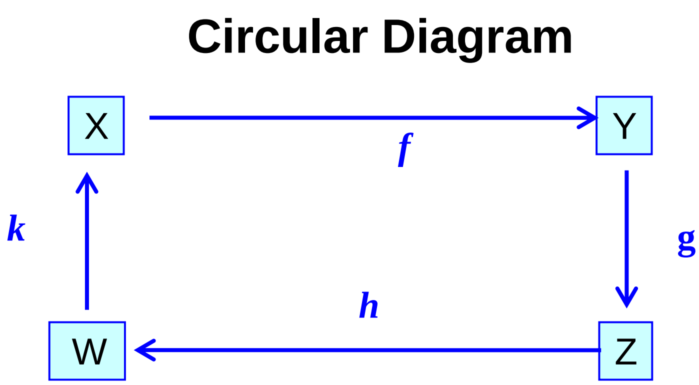


Figure 7 – A diagram is called circular if the composition of its morphisms yields the identity morphism

Several examples of systems satisfying the circularity condition are now given.

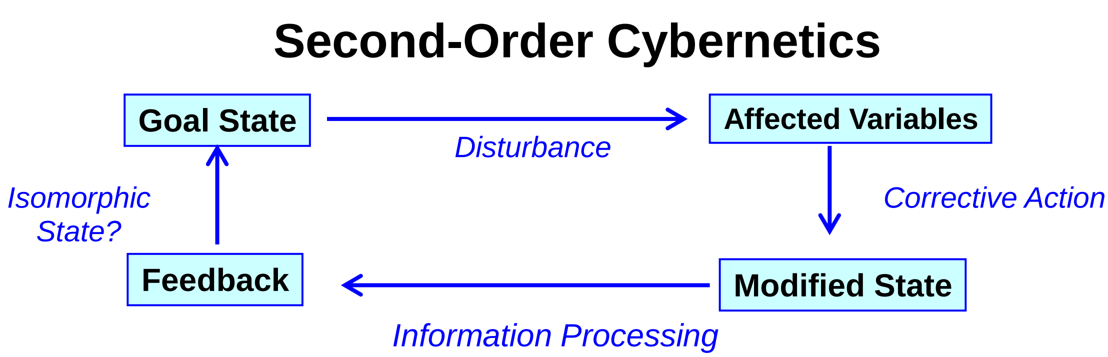


Figure 8 – Feedback cycle of components of a control system

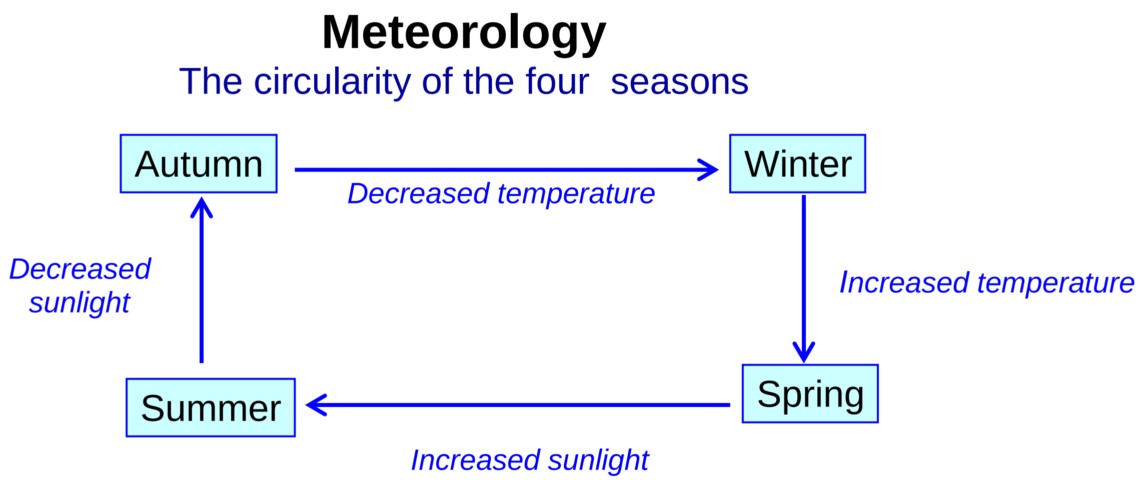


Figure 9 – The seasonal weather fluctuation is a circular diagram

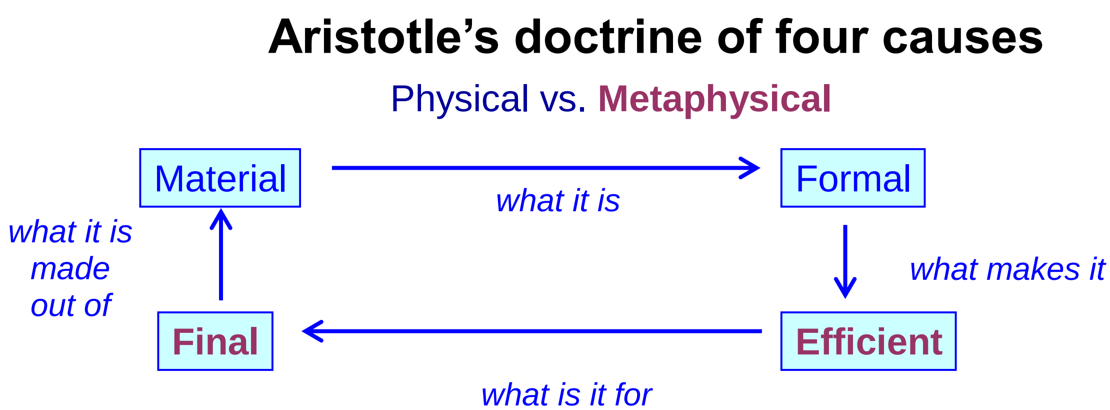


Figure 10 – Aristotle's conception of causality is a circular diagram

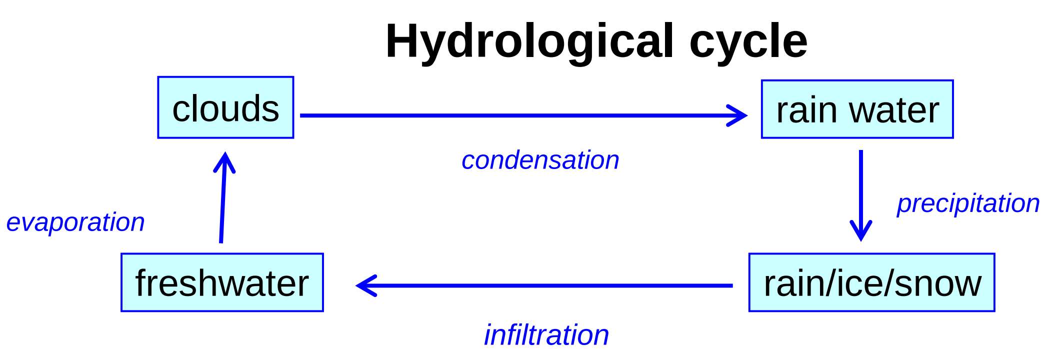


Figure 11 – The cycle of the earth's water distribution

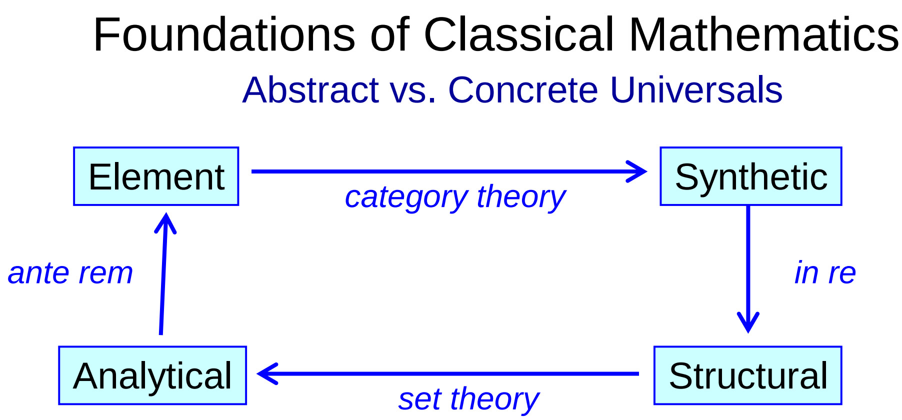


Figure 12 – Attempts to formalize classical mathematics into an axiomatic form leads to a circular diagram

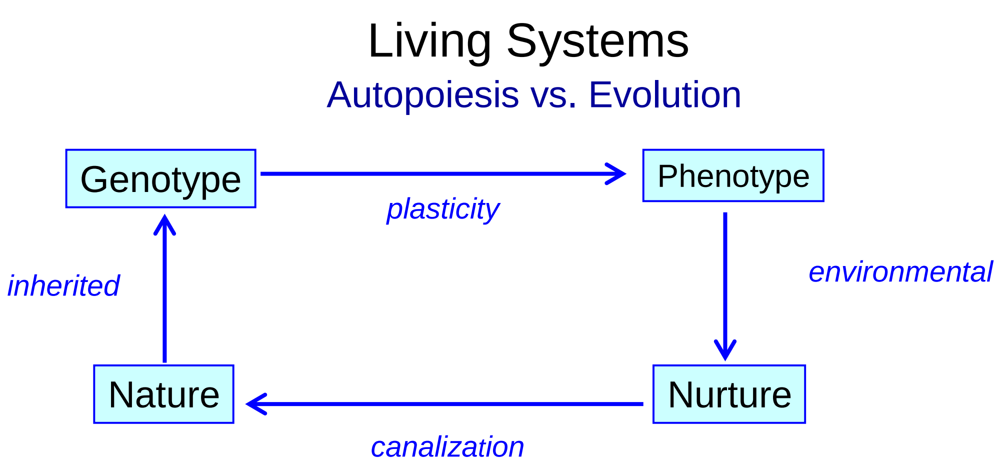


Figure 13 – The complementarity between function and structure in living systems produces a circular diagram

Incompleteness, inconsistencies, paradoxes, self-references, and circular arguments have been obstacles in establishing a sound and logical basis for mathematical and scientific reasoning [2], [12], [13], [19], [43]. Duality, on the other hand, has been a useful construct in analyzing both the physical and metaphysical world. Duality has been studied in chemical [30], biological [32], atomic [35], ecological, and sociological systems as well as in axiomatic systems like algebraic geometry and topology. It is a fundamental concept in the natural and social sciences as well as in the humanities. Also fundamental to the duality concept is the role of *circularity, circular arguments,* and *self-referential systems*.

A major reason that neither biology nor chemistry is classified as a deductive science is that concepts, very often, are defined in a circular manner. In biology, it is impossible to explain what *life* is without a reference to the complementary concept of *death*. In chemistry, it is also impossible to define what an *acid* is without reference to the dual concept of a *base*. Some scientists define chemistry as the material science whose method is to *separate* (dissociation, decomposition, cleavage, homolysis and heterolysis) and to *unite* (condensation, addition, self-assembly, binding and bonding, association, aggregation, coagulation, synthesis, combination) [24]. Other examples of dualistic terminology in chemistry highlight the reason for this definition: oxidant/anti-oxidant developed from the observation of electron transfer between an oxidant and a reducer, stability/instability of energy levels, inert/labile, electrophile/nucleophile pair, substrate/reagent, host/guest, receptor/agonist. It is many times impossible to tell from the paired terms that a duality or complementarity relationship exists. This state of affairs is true for almost every academic discipline: a phletora of esoteric terminology and proliferation of confusing terminology resulting.

It is important to recall that many of the definitions in modern chemistry come from alchemy that is itself based upon dualistic classifications [25]. Matter was envisaged by the alchemists as bipolar, organized by a tension between two opposing and, oftentimes, complementary principles. Ancient Chinese alchemists organized minerals and substances into lists, according to their yin or yang characterization. A treatise, written between the 3rd and the 7th century, lists cinnabar and realgar as yang type, while mercury and orpiment were considered as being of the yin type [26].

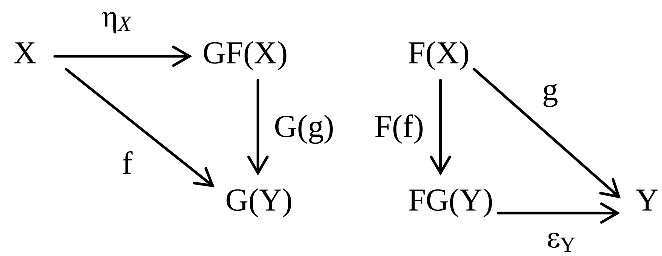
7 DUALOGY

Before presenting the main propositions of Dualogy it is important to note that circularity and feedback play an important role in control theory, both mechanical and biological. An interpretation of metabolic networks, called *(M,R)* systems where M=metabolism and R=repair, is an attempt to prove that metabolic closure (or metabolic circularity) could be explained in purely formal terms [33]. The central motto of the research program can be traced to an insight on the nature of cellular metabolic networks [34]; this consists of a semi-formal method to explain how the network of biochemical processes that constitutes metabolism bootstraps itself without the help of external agents generated outside the network. The cell organization, i.e. its functions, remains invariant in spite of continuous structural changes. A large part of the development of these ideas is based on category theory. In mathematical terms, the major insight appears as a result about sets (structure) and admissible transformations (functors) between them.

Models that have been defined in terms of categories, functors and natural transformations include unimolecular chemical transformations, multi-molecular chemical, and biochemical transformations. Applications of such natural transformations are then utilized to analyze protein biosynthesis, embryogenesis and nuclear transplant experiments. Complex graph matching problems, treated as a combinatorial optimization problem, are also amenable to category theory. They include computational complexity, neural networks, linear programming, weighted graph matching, quadratic optimization, simplex-based algorithm, Hungarian method, eigendecomposition, pattern recognition, symmetric polynomial transform, genetic algorithms, probabilistic relaxation, and clustering techniques. Genetic and combinatorial information can be recorded as a bond graph and the topology of interconnection of the components analyzed. The cycle structure can be concisely represented by a matroid and its dual [6].

All pairs of adjoint functors arise from universal constructions. If we let *F* and *G* represent a pair of adjoint functors with unit η and co-unit ε, then we have a universal morphism for each object in *C* and *D*:

- For each object *X* in *C*, ($F(X)$, η_X) is a universal morphism from *X* to *G*. That is, for all $f : X \longrightarrow G(Y)$ there exists a unique $g : F(X) \rightarrow Y$ for which the following diagrams commute.
- For each object *Y* in *D*, ($G(Y)$, ε_Y) is a universal morphism from *F* to *Y*. That is, for all $g : F(X) \longrightarrow Y$ there exists a unique $f : X \longrightarrow G(Y)$ for which the following diagrams commute.



There exists yet another characterization of adjoint functors via the unit η: $1_C \rightarrow GF$ and co-unit ε: $FG \rightarrow 1_D$. These natural transformations have the following properties: the composition (εF) o ($F\eta$), a natural transformation $F \rightarrow FGF \rightarrow F$, is equal to 1_F, and the composition ($G\varepsilon$) o (ηG): $G \rightarrow GFG \rightarrow G$ is equal to 1_G. Conversely, given any two natural transformations η and ε having these properties, we say that the functors *F* and *G* form an adjoint pair. Adjoints are considered as *generalized inverses*. In category theory every statement, theorem, or definition has a dual that is obtained by "reversing all the arrows." If one statement is true in a category *C* then its dual will be true in the dual category C^{op}. Adjoint functors map in opposite directions and such a pairs of functors typically arise from a construction defined by a universal property.

The four meteorological seasons can be classified as two pairs of adjoint functors that satisfy the circularity condition over time. They are non-commutative. They follow a specific cyclical order. Non-commutative elements are evident in mathematical physics, space-time coordinates (x,y,z,t), and genetics. The French mathematician and philosopher Descartes merged algebra and Euclidean geometry. This work was influential in the development of analytic geometry, calculus, and cartography. The Cartesian coordinate-system is defined by two axes, constructed at right angles, forming the *xy*-plane. We can represent the imaginary numbers on the vertical axis of the complex number plane. Imaginary numbers are very useful in describing circuits in electronics and electrical engineering. Attempts to extend imaginary numbers to three dimensions failed but in 1843 Sir. William R. Hamilton successfully extended imaginary numbers to four dimensions calling them *quarternions*. Quaternions are non-commutative extension of the complex numbers. In 1864 Maxwell used imaginary numbers in the equations of electromagnetism and were written in the form of complex numbers and vectors. Since the unit quaternions form the Lie Group Sp(1) = SU(2) =

Spin(3) = S3, Maxwell's use of quaternions in electromagnetism anticipated the SU(2) weak force and the SU(2)XU(1) electroweak unification, and Maxwell's consideration of a *compressible general elastic aether medium* anticipated the Higgs mechanism and torsion physics.

In physics the quaternions play a role similar to base pairs in genetics. In the science of heredity two nucleotides on opposite complementary DNA or RNA strands that are connected via hydrogen bonds are called a base pair. DNA is a chain of chemical *building blocks*, called *bases*, of which there are four types that are abbreviated: A, T, C, and G. Each base can only *pair up* with one single predetermined other base: A+T, T+A, C+G and G+C are the only possible combinations; that is, an *A* on one strand of double-stranded DNA will *mate* properly only with a *T* on the other, complementary strand. Because each strand of DNA has directionality, the sequence order does matter: A+T is not the same as T+A, just as C+G is not the same as G+C. Base pairing is non-commutative under addition.

The four seasons, Aristotle’s four causes, the base pairs in DNA, the quaternions, and the duality of four dimensional space-time proposed in theoretical particle physics to explain structure are the justification for the Propositions of Dualogy stated below. The formal rules of category theory come from mathematics and significant applications can be found in computer science, biology, and genetic algorithms (an algorithm for optimizing a property based on an evolutionary mechanism that uses replication, deletion, and mutation processes carried out over many generations). The following propositions are a summary of the preceeding analysis of duality from the perspective of category theory.

Proposition I: All phenomena in creation, material and immaterial, exist in a dual state. In order to describe any single event, action, state of being, thought, or any other real or abstract concept it is necessary to identify two distinct complementary co-dependent entities that define the state (conjugate variables) and behavior (conjugate transformations) of the phenomenon under consideration.

Proposition II: Conjugate variables, C1 and C2, and conjugate transformations, T1 and T2, that satisfy the duality properties stated in Proposition I also satisfy generalized conservation principles.

Proposition III: Dualities obey the rules of commutative diagrams in which the conjugate transformations act as the adjoint functors.

Proposition IV: Diagrams that satisfy the circularity condition can be converted into commutative diagrams and vice versa.

Proof: Given any four morphisms, f, g, h, and k that satisfy the circularity condition, $\mathbf{k \circ h \circ g \circ f = 1}$, whose inverses exist, it follows that $\mathbf{g \circ f = h^{-1} \circ k^{-1}}$

Proof: Since $\mathbf{k \circ h \circ g \circ f = 1}$, we can apply the inverses of k and h successively to each side of the equation to produce the desired result that will produce the commutative diagram.

Example: Letting f = Material, g = Formal, h = Efficient, and k = Final we can conclude that

$$\text{Formal} \circ \text{Material} = (\text{Efficient})^{-1} \circ (\text{Final})^{-1}$$

Proposition V: Complex and/or hierarchical structures are created from a pair of dualities that satisfy the circularity condition. This proposition, whose validity will be demonstrated in a forthcoming paper, follows from recent developments in genetics [40], particle physics [42], and the taxonomy of the dynamics of nonlinear systems [39].

CONCLUSION

Many advantages and benefits are derived from using category theory across the disciplines. Category theory, notwithstanding its highly abstract nature,

- is a **unifying language** for discussing different mathematical models and other logic-based structures;
- reveals **common structures** in seemingly unrelated systems and a framework for comparing them,
- reveals **invertible structures**, i.e. for every categorical construct there is a dual formed by reversing all the transformations,
- consolidates the description of similar operations such as **'products'** found in set theory, group theory, linear algebra, and topology, and
- produces **graphical models** which are intuitive, formal, declarative, and subject to further analysis.

Another major advantage derived from the application of category theory to multiple disciplines that incorporate a principle of duality, dual variables, dual transformations, or other dualistic constructs into the formulations of their theoretical or empirical models is that terminology can be standardized across the curriculum. Since 1978, on the average, over forty new scientific journals per week have been published – a true explosion of information. The work of scientists and mathematicians has been interdependent throughout the centuries. Standardizing terminology across the disciplines in which duality concepts exist should become the new priority in the curriculum mandate. The humble contribution of this paper is an effort in that direction.

REFERENCES

[1] Andrade, E. In: The Interrelations between Genotype/Phenotype/Environment: A Semiotic Contribution to the Evo:Devo Debate. http://www.library.utoronto.ca/see/SEED/Vol5-2/Andrade.pdf. Cited 23 May 2007 (2007)

[2] Awodey, S.: Structure in Mathematics and Logic: A Categorical Perspective. Philosophia Mathematica 4, 209-237 (1996)

[3] Baez, J., Categorification. http://arxiv.org/PS_cache/math/pdf/9802/9802029v1.pdf Cited 23 May 2007 (2007)

[4] Baianu, I.: Organismic Supercategories and Qualitative Dynamics of Systems. Bull. Math. Biophys., 33, 339-354 (1971)

[5] Barr, M., Wells, C.: Category Theory for Computing Science, Montreal, CRM (1999)

[6] Birkett, S., Roe, P.H. In: The Mathematical Foundations of Bond Graphs-III. Matroid Theory. http://real.uwaterloo.ca/~sbirkett/bondgraphs/mfbg3.pdf. Cited 23 May 2007 (2007)

[7] Breedveld, P. C.: In: Modeling and simulation of dynamic systems using bond graphs. Control Systems, Robotics and Automation, from Encyclopedia of Life Support Systems (EOLSS), Developed under the Auspices of the UNESCO, Eolss Publishers, Oxford, UK. http://www.eolss.net. Cited 23 May 2007 (2007)

[8] Cheever, E.: In: Analogous Electrical and Mechanical Systems. http://www.swarthmore.edu/NatSci/echeeve1/Ref/Analogs/ElectricalMechanicalAnalogs.html Cited 23 May 2007 (2007)

[9] Cheon, T., Shigehara, T.: Fermion-Boson Duality of One-Dimensional Quantum Particles with Generalized Contact Interactions. Phys. Rev. Lett. 82, 2536-2539 (1999)

[10] Crane, L., Frenkel, I: Four dimensional topological quantum field theory, Hopf categories, and the canonical bases, Jour. Math. Phys. 35, 5136-5154 (1994)

[11] Dahl, B. In: A Synthesis of Different Psychological Learning Theories? - Piaget and Vygotsky. http://www.people.ex.ac.uk/PErnest/pome17/pdf/bdahl.pdf. Cited 23 May 2007 (2007)

[12] Drossos, C. A. In: Sets, Categories and Structuralism http://www.math.upatras.gr/~cdrossos/Docs/Dros4WICT.pdf Cited 23 May 2007 (2007)

[13] Drossos, C. A.: Structures, Points, and Levels of Reality. G. Sica (Ed.) Essays on the Foundations of Mathematics and Logic, pp. 83-114, Polimetrica International Scientific, Monza, Italy (2005)

[14] Eco, U.: Semiotics and the Philosophy of Language. Indiana University Press. Bloomington, IN (1984).

[15] Eilenberg, S., Mac Lane, S.: General Theory of Natural Equivalences, Transactions of the American Mathematical Society, 58, 231-294 (1945)

[16] Englert, B.: Duality Condition. Phys. Rev. Lett., 77, 2154 (1996)

[17] Farhi, E., Gutmann, S.: Int. J. Mod. Phys. 5, 3029 (1990)

[18] Gao, D. Y.: Duality Principles in Nonconvex Systems. Theory, Methods, and Applications. Kluwer Academic Publishers, Dordrecht (now Springer) 1999.

[19] Godel, K.: On Formally Undecidable Propositions of Principia Mathematica and Related Systems. New York Basic Books (1962 translation).

[20] Healy, M. J., Caudell, T. P.: Neural Networks, Knowledge, and Cognition: A Mathematical Semantic Model Based upon Category Theory. UNM Technical Report: EECE-TR-04-020 (2004)

[21] Heylighen, F., Joslyn C. In: Cybernetics and Second-Order Cybernetics, R.A. Meyers (ed.), Encyclopedia of Physical Science & Technology (3rd ed.). Academic Press, New York, (2001). http://pespmc1.vub.ac.be/Papers/Cybernetics-EPST.pdf. Cited 23 May 2007 (2007)

[22] Hilbert, D.: Über die Grundlagen der Geometrie, Math. Ann. 56, 381-422 (1903)

[23] Kato, G.: Cohomology, Precohomology, Limits and Self-similarity of Conscious Entity (Sheaf Theoretic and Categorical Formulation of Consciousness), *Noesis*, 26, 47-55 (2001)

[24] Laszlo, P.: *La leçon de choses*. Paris, Austral. (1995)

[25] Laszlo, P.: Qu'est-ce que l'alchimie? Questions de science. Paris, Hachette. (1996) http://www.ulg.ac.be/lcfi/laszlo/concepts.html

[26] Laszlo, P.: Molecular Correlates of Biological Concepts. Comprehensive Biochemistry. Amsterdam: Elsevier. 69-71 (1986)

[27] Mazzola, G.: The Topos of Music. BirkhaÄuser, Verlag (2002)

[28] Macnamara J., Reyes, G. E.: The Logical Foundation of Cognition. Oxford Univ. Press (1994).

[29] Markopoulou, F.: In: The internal description of a causal set: What the universe looks like from the inside. 1999 http://arxiv.org/abs/physics/0605107 Cited 23 May 2007 (2007)

[30] Melhado, E. M.: Jacob Berzelius, The Emergence of his Chemical System. The University of Wisconsin Press. Madison, WI (1981)

[31] Mitra, A. N. In: Duality : A Bridge Between Physics And Philosophy? http://arxiv.org/abs/physics/0605107 Cited 23 May 2007 (2007)

[32] Popper, K. R.: Studies in the Philosophy of Biology. University of California Press, Berkeley, CA, 259-284 (1974).

[33] Rosen, R.: Life Itself. Columbia University Press, New York (1991)

[34] Rosen, R.: The representation of biological systems from the standpoint of the theory of categories Bull. Mathematical Biophysics, 20, 317-341 (1958),

[35] Santilli, R. M.: Lie-Admissible Approach to the Hadronic Structure. Hadronic Press. Nonantum, MA. 1, 1-30 (1978).

[36] Shoaf, R. A.: Milton, Poet of Duality: A Study of Semiosis in the Poetry and the Prose. Yale University Press. New Haven, CT. 1-10 (1985).

[37] Sfard, A. On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin, Educational Studies in Mathematics, 22 (1), 1–36 (1991)

[38] Tufillaro, N.B., Reilly, J. In: An experimental approach to nonlinear dynamics and chaos. Online book: http://cnls.lanl.gov/~nbt/Book/node1.html Cited 23 May 2007 (2007)

[39] Venkatasubramanian, V., Schattler, H., Zaborszky, J.: Dynamics of Large Constrained Nonlinear Systems –A Taxonomy Theory. Proceedings of the IEEE, November, Vol. 83, No. 11, 1530--1561 (1995)

[40] von Dassow, Meir, G., E., Munro, E. M., Odell, G. M.: The Segment Polarity Network is a Robust Developmental Module. Nature 406, 188-92 (2000)

[41] von Dassow, G., Odell, G. M.: Architecture and Constraints of the *Drosophila* Segment Polarity Module: Robust Spatial Patterning Emerges from Intertwined Cell State Switches. J. Exp. Zool. (Mol. Dev. Evol.) 294, 179-215 (2002)

[42] Yershov, V. In: Dualism of spacetime as the origin of the fermion mass hierarchy. Apeiron, Vol. 12, No. 1, January 2005 http://redshift.vif.com/JournalFiles/V12NO1PDF/V12N1YER.pdf Cited 23 May 2007 (2007)

[43] Zhang, J. In: Referential Duality and Representational Duality in the Scaling of Multi-Dimensional and Infinite-Dimensional Stimulus Space 2004 http://www-personal.umich.edu/~junz/madrid012705.pdf Cited 23 May 2007 (2007)